Logical Argumentation: A Tutorial

The 6th Summer School on Argumentation, 2024, Hagen

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In a nutshell

Logical argumentation:

- modelling **defeasibility of reasoning** through the interaction of arguments and counter-arguments,
- where arguments are complex **premise-conclusion structures generated by a logic**.

In fact, logical argumentation is a **unifying framework** for the representation, comparison, and study of **nonmonotonic logics!**

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In fact, logical argumentation is a **unifying framework** for the representation, comparison, and study of **nonmonotonic logics!**

We dive into this in the next 3,5 hours.

Outline

The outline for today:

1. Defeasible reasoning and NML
2. Formal argumentation
3. Logical argumentation
4. Metatheory, properties, and desiderata
5. Application to normative reasoning

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break
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Part 1: Defeasible Reasoning

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First, **reasoning** is drawing conclusions from assumptions using inference rules:

- **Deductive**: making inferences that hold without exception.
- **Defeasible**: retain the option to **retract an inference.**

E.g., upon hearing

> IT HAS BEEN TOLD that princess Charlotte killed the dragon Norbert . . .

what conclusions would we draw?

- Charlotte is a skilled fighter (since dragons are large and dangerous).
- Charlotte really did kill Norbert.

Part 1: defeasible reasoning

When reasoning defeasibly **we jump** to conclusions:

- Nobody said that Norbert is large and dangerous...

> THE LOCAL DRAGON PROTECTION GUILD WAS OUTRAGED. Since years they have been lobbying at the king's court that baby dragons are not to be admitted to show fights with the royal offspring.

Upon learning the above, **we surely want to retract** some inferences!

- Norbert was just a baby, Charlotte was not necessarily skilled.

Part 1: defeasible reasoning

When reasoning defeasibly, the given information is **uncertain**:

- Information may turn out to be incorrect or may be disputed.

> THE ROYAL PROPAGANDISTS PLANTED THE STORY of the princess' brave killing in all the royal news outlets, while in reality poor Norbert died of old age.

Upon learning the above, we may also **want to retract inferences:**

- Norbert was not killed at all!

Part 1: defeasible reasoning

Defeasible reasoning is not just a curiosity!

Yes, deductive reasoning is pivotal to mathematics and science.

> But if we were only to reason deductively on certain information, we wouldn't come far on a daily base.

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Nonmonotonic logics (NML):

- **Formal approach** to defeasible reasoning.

Part 1: monotonic logic

Most logics in **traditional logic** text books are **monotonic**:

- Classical logic (CL), intuitionistic logic, . . .
- Monotonic inferences are absolutely warranted, **without exception**.

E.g., let $\vdash_{\mathsf{CL}}$ be classical entailment:

$$\phi \wedge \psi \vdash_{\mathsf{CL}} \phi$$

$$\theta, \phi \wedge \psi \vdash_{\mathsf{CL}} \phi$$

$$\theta \to \bot, \theta, \phi \wedge \psi \vdash_{\mathsf{CL}} \phi$$

$$\ldots$$

CL **preserves the truth/derivability** throughout the inference process:

- Conclusions accumulate and are never retracted

Part 1: nonmonotonic logic

For nonmonotonic logics monotonicity does not hold (deliberately!):

$$\text{e.g., } \phi \vdash \phi \text{ and } \phi, \neg\phi \nvdash \phi$$

- Great for **reasoning with incomplete, uncertain, and inconsistent knowledge bases**.

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À branch of Artificial Intelligence central to **Knowledge Representation and Reasoning**:

- Default logic (Reiter, 1980);
- Autoepistemic logic (Moore, 1985);
- KLM approach (Kraus et al, 1990) and (Lehman and Magidor, 1992);
- Input/Output logic (Makinson and van der Torre, 2001);

  . . .

Ah, but!

The investigation of defeasible reasoning **goes much further back**:

- Aristotle (384 – 322 BCE) distinguished deductive from dialectic (defeasible) reasoning.
- Ross (1930) argued that moral reasoning is defeasible. Duties are prima facie: 'You should not lie' is by default, not absolute.
- Hart (1948) introduced the term 'defeasibility' in the context of legal contracts.
- **Toulmin (1958) explicitly attacked formal logic** (CL at the time) for its inability to reason defeasibly!

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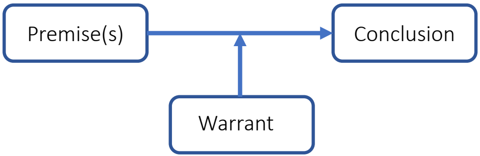

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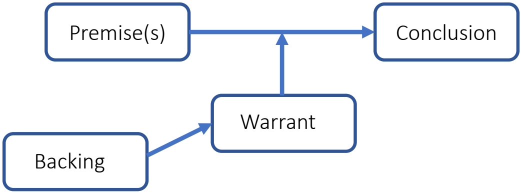

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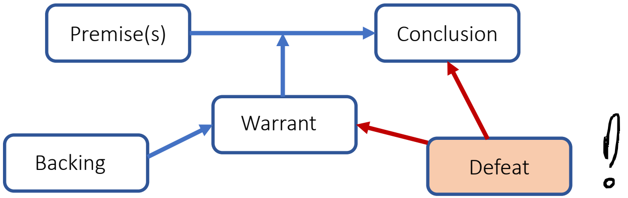

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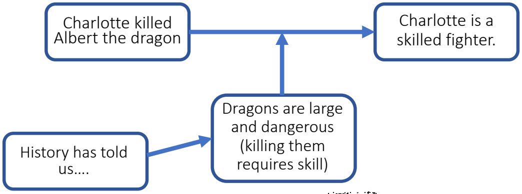

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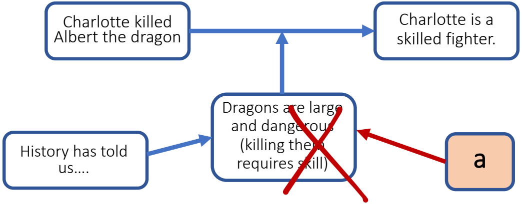

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- a Albert was just a baby!
- b Albert was sleeping and Charlotte got help. . .
- c Journalists discovered the killing was a propagated fake story.

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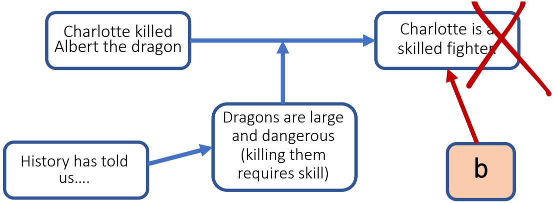

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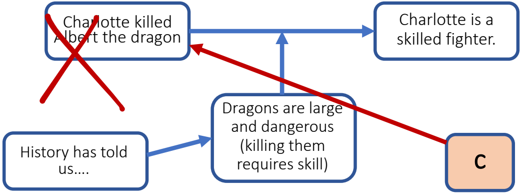

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Toulmin provided the foundation for:

- structural representation of arguments;
- and the analysis of their defeasibility through counter-arguments.

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- structural representation of arguments;
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The birth of (semi)-**formal argumentation** as a defeasible reasoning framework.

> And nowadays, **formal argumentation** is a **uniform framework** for NMLs!

...

Logical Argumentation for Defeasible Reasoning

Tutorial at COMMA Summer School 2024, Hagen

Kees van Berkel and Christian Straßer

September 13, 2024

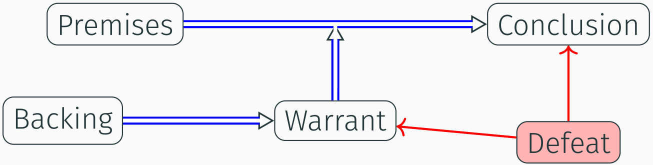
Ruhr University Bochum

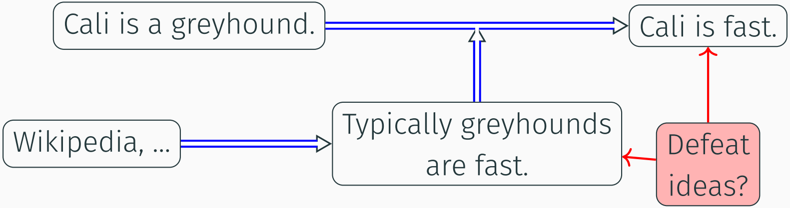
Outline

Part 2. Warming Up

Toulmin [18] challenged classical (monotonic) logic by pointing out that most everyday inferences are subject to defeat: they are defeasible.

Toulmin's take on nonmonotonic inference was an argumentative, informal one.





When designing a formal model, it’s good to get inspiration from natural examples, so let’s start with one!

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Argument ***a*.** Charlotte is a brave princess, since she killed Norbert.

THESE DAYS it is considered especially brave for the royal offspring to kill dragons. Princess Charlotte killed the dragon Norbert. Charlotte also led the expedition into the dungeons of the underworld. The underworld is a dangerous place and it takes guts to enter it.

Argument *a*. Charlotte is a brave princess, since she killed Norbert.

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Argument *c*. Charlotte cannot be claimed to be brave based on her killing poor Norbert, since Norbert is just a baby dragon.

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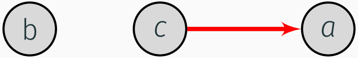
Central Idea. We abstract away from content and track conflicts with diagrams!



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A rebut goes for the conclusion.

In order for formal argumentation to offer a useful model of defeasible reasoning it needs to

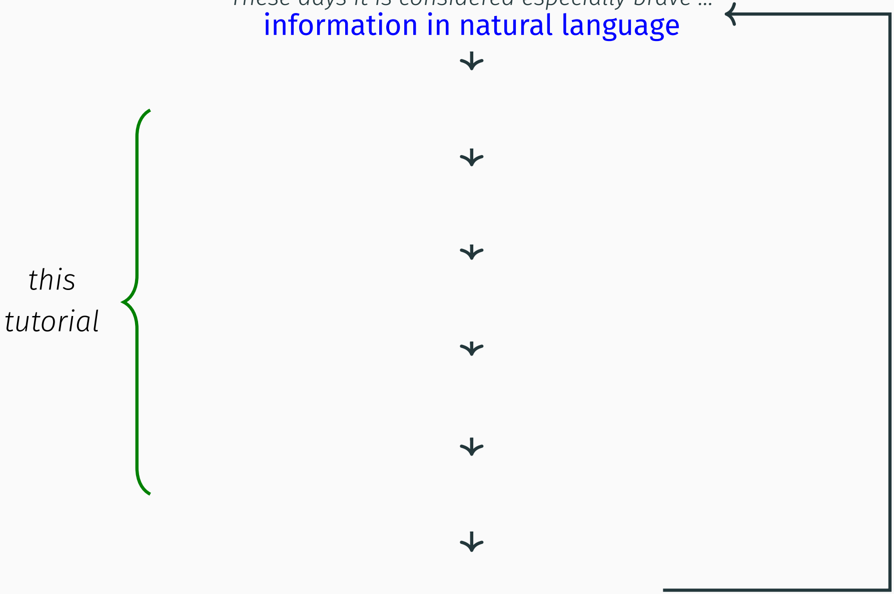
- provide structure to arguments
- track different types of attacks and in this way track conflicts,
- and to indicate when to retract inferences.

... an (extended) ArgKRR-pipeline for defeasible argumentative reasoning ... [1]

[1] ArgKRR is our term for argumentative knowledge representation and reasoning. KRR exists, ArgKRR we made up. The idea of describing the approach underlying formal argumentation as a pipeline is taken from Martin Caminada's work ...

These days it is considered especially brave …

information in natural language



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now →

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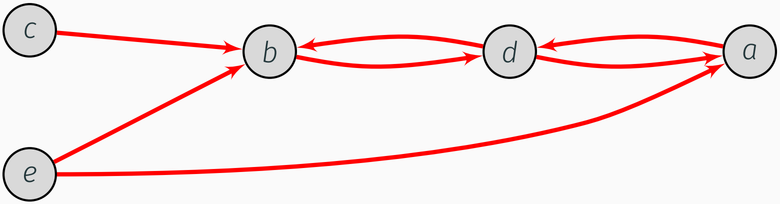
Argumentation semantics

In his [10], Dung defined argumentation semantics. They give ways to select arguments from an argumentation framework, such as



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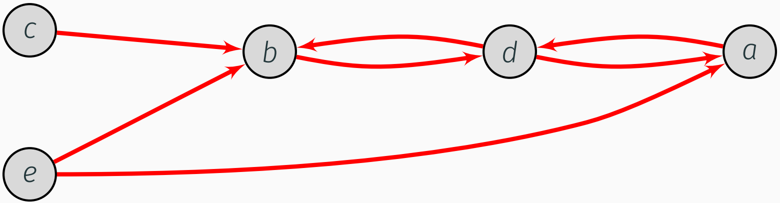


Definition

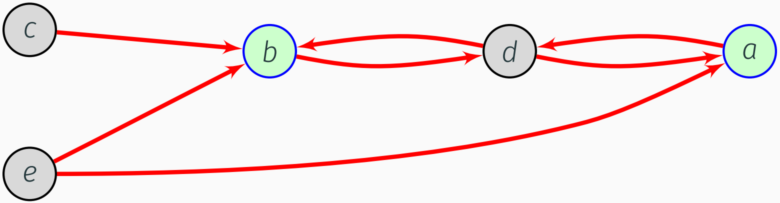
An argumentation framework (in short, AF) is nothing but a directed graph ⟨Args, Attack⟩ with Args representing arguments and Attack representing argumentative attacks.

What are good criteria to select arguments in $\langle \text{Args}, \text{Attack} \rangle$?

A very basic criterion is to select arguments that don't conflict with one another.

Definition

A set of arguments $\mathcal{A} \subseteq \text{Args}$ is conflict-free if there a no $a, b \in \mathcal{A}$ such that $(a, b) \in \text{Attack}$.



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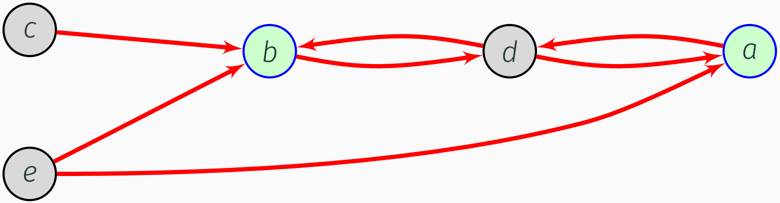
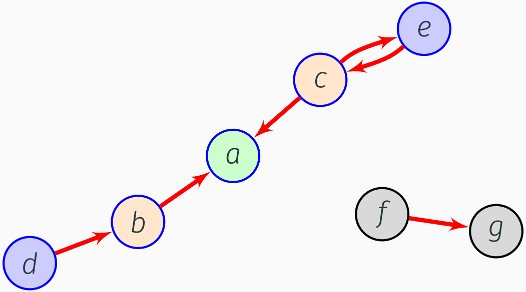


This is not really satisfying …

Definition

A set $\mathcal{A}$ defends and argument a if it attacks every attacker of a.

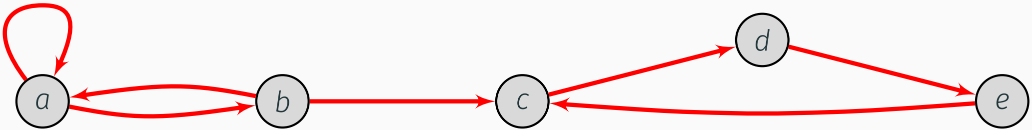


Definition

$\mathcal{A}$ is admissible if it defends every of its arguments and it is conflict-free.

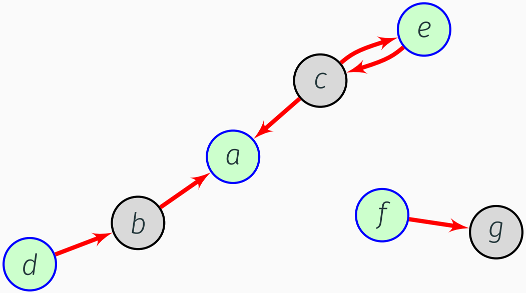
Exercise: Admissible sets

What are admissible sets in the following AF? How many are there?



Definition

$\mathcal{A}$ is complete if it is admissible and it contains all the argument it defends.



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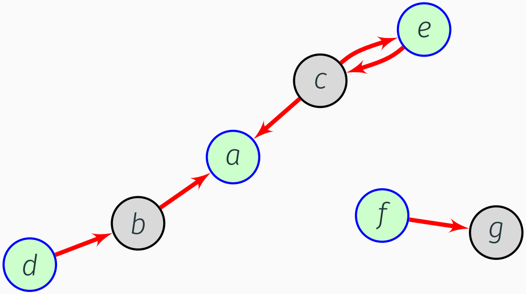
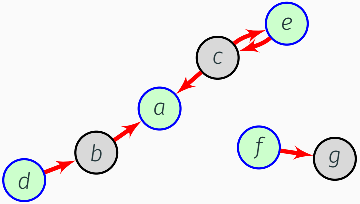


Do you see other complete extensions? How many can you find?

Complete extensions: examples



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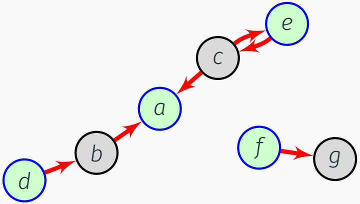

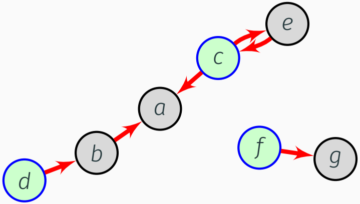

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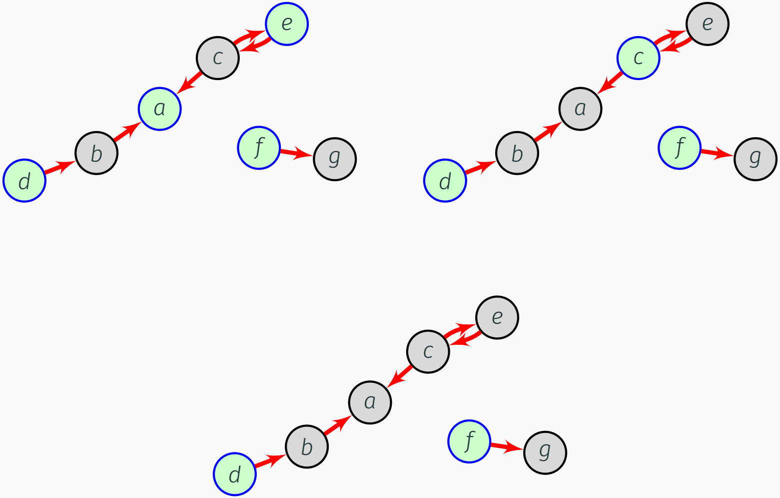
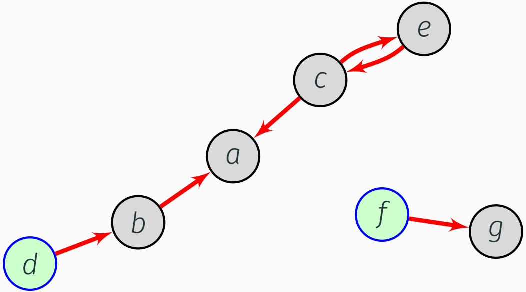


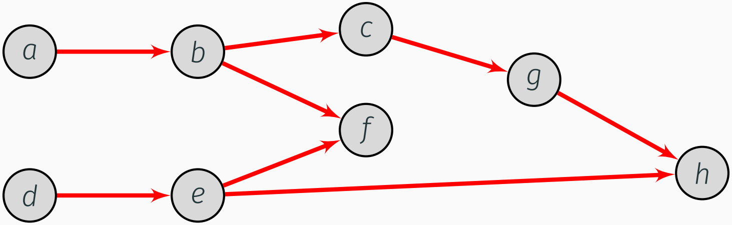
Definition

$\mathcal{A}$ is grounded if it is the unique $\subset$-smallest complete set.

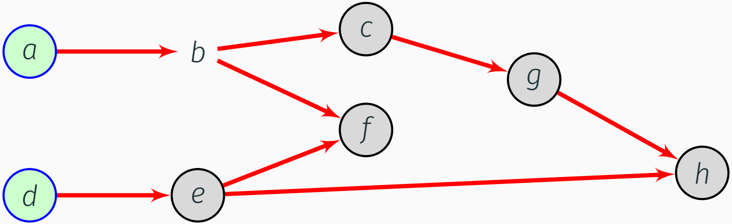


Exercise: grounded

What is the grounded extension in the following framework? How many arguments does it contain?



Simple algorithm to find grounded extension: illustrated



Algorithm for finding the grounded set $\mathcal{G}$ in a finite AF. Let $\mathcal{G}^\star = \emptyset$ and loop:

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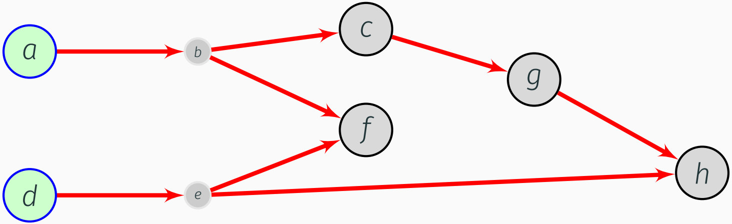

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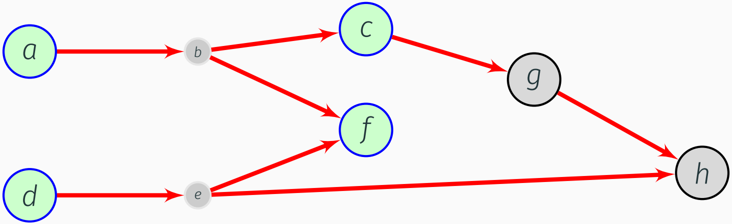
Simple algorithm to find grounded extension: illustrated



Algorithm for finding the grounded set $\mathcal{G}$ in a finite AF. Let $\mathcal{G}^\star = \emptyset$ and loop:

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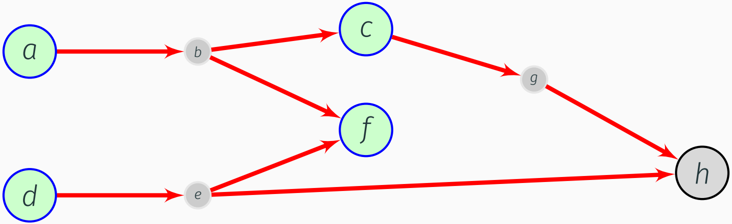

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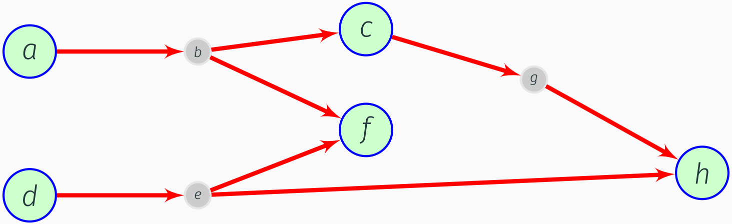

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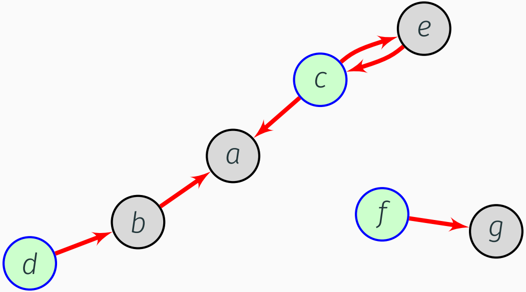
Definition

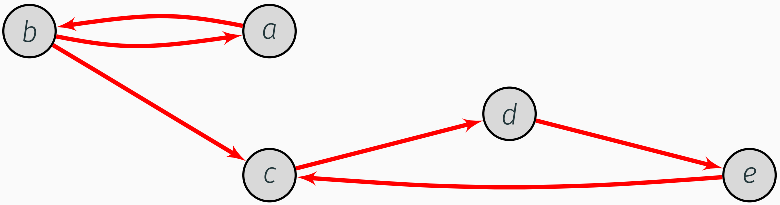
$\mathcal{A}$ is stable if it is conflict-free and it attacks every argument that is not in $\mathcal{A}$.



Do you see another one?

Exercise: stable extensions

What are stable sets in the following AF? How many can you find?



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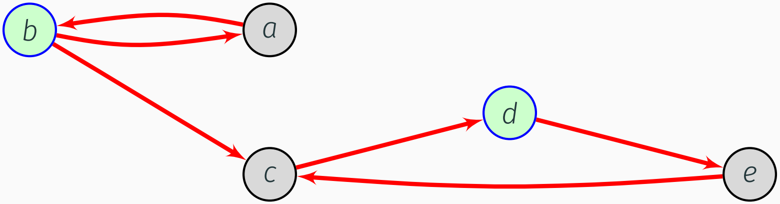

Other semantics

There are many other argumentation semantics. For instance, instead of selecting the minimal complete set, one can be interested in maximal complete sets (preferred semantics). Other semantics were specifically designed to deal with odd attack cycles (e.g., self-attacking arguments, etc.) or explanations. For an overview see [7].

Part 3. Logical Argumentation

These days it is considered especially brave ...

information in natural language

↓

$\mathbb{K} = \{$💀$_{\text{Cha}}(\text{Nor}) \supset \texttt{brave}(\text{Cha}), \ldots\}$

information in formal language

↓

💀$_{\text{Cha}}(\text{Nor})$, 💀$_{\text{Cha}}(\text{Nor}) \supset \texttt{brave}(\text{Cha}) \Rightarrow \texttt{brave}(\text{Cha})$

generate arguments and attacks (logic)

↓

✔ $a \to b \to c$

argumentation framework

↓

✔ $a \to b \to c$

select arguments via semantics

↓

$\mathbb{K} \mid\!\sim \phi$

determine consequences

↓

Why does ϕ follow, rather than ψ?

explain consequences

Let's start with a story ...

CHARLOTTE fought at the great battle with the dragon kingdom. Two fierce dragons were particularly frightening, the twins Norbert and Albert. At the victory dinner, Charlotte's sister boasted that Charlotte killed the dragon Norbert, not Albert. In contrast, Charlotte's brother claims that she killed Albert, not Norbert. It takes a brave person to kill a dragon.

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Let us translate this into a formal language.

In the following we will us a simple language, including the usual connectives $\neg, \wedge, \vee, \supset$.

Knowledge bases

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Definition

A knowledge base $\mathbb{K}$ is then given by:

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We will make a distinction between two types of assumptions:

strict assumptions $\mathcal{A}_s$. these we consider to hold certainly

defeasible assumptions $\mathcal{A}_d$. these hold by default, but given good reasons, we may give up on them

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Our strict assumptions $\mathcal{A}_s$ are given by:

$$\mathcal{A}_s = \{ \text{📢}_{\text{sis}} n, \ \text{📢}_{\text{bro}} a, \ \text{☠}_{\text{Cha}}(\text{Nor}) \supset \text{brave}(\text{Cha}), \ \text{☠}_{\text{Cha}}(\text{Alb}) \supset \text{brave}(\text{Cha}) \},$$

where (skipping the subscript in "☠_{Cha}")

$$n = \text{☠}(\text{Nor}) \wedge \neg \text{☠}(\text{Alb})$$
$$a = \text{☠}(\text{Alb}) \wedge \neg \text{☠}(\text{Nor})$$

What about defeasible assumptions for modeling our story?

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What somebody claims is not always certainly the case, but (given a charitable interpretation) it is so by default. We model this by means of defeasible assumptions.

Definition

In the following our defeasible assumptions are given by:

$$\mathcal{A}_d \quad = \quad \{ 📢_{\text{agent}} \phi \supset \phi \mid \phi \in \mathcal{L} \}$$

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We let:

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Read: agent is a reliably source concerning ϕ

and so

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!

Note that, by Modus Ponens, n follows from $📢^{☑}_{\text{sis}}n$ and $📢_{\text{sis}}n$.

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explain consequences

How to generate arguments?

Formal arguments

- Arguments conclude something on the basis of some assumptions (or premises). We write:

$$\phi_1, \ldots, \phi_n \Rightarrow \psi$$

where $\phi_1, \ldots, \phi_n \in \mathbb{K}$ are assumptions in our knowledge base.

Formal arguments

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- therefore Charlotte killed Norbert but not Albert (n).
- Arguments have the form of sequents:

$$\psi_1, \ldots, \psi_n \Rightarrow \phi.$$

But, how to generate arguments?

The place for logic

We use logic for that! Paradigmatically we will stick to classical logic for this tutorial.

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$$\phi_1, \ldots, \phi_n \quad \Rightarrow \quad \psi$$

iff

ψ follows by classical logic from $\{\phi_1, \ldots, \phi_n\}$. (In signs: $\{\phi_1, \ldots, \phi_n\} \vdash \psi$)

How to derive arguments?

We can use proof theory. There are many options:

- Hilbert style
- Natural deduction
- Semantic tableaux
- Sequent calculi

How to derive arguments?

Sequent calculi seem especially interesting when dealing with arguments, since they allow to construct new arguments from arguments by allowing to manipulate both the premises and the conclusion of an argument. For instance,

$$\frac{\phi_1, \phi_2, \ldots, \phi_n \Rightarrow \psi}{\phi_1 \wedge \phi_2, \phi_3, \ldots, \phi_n \Rightarrow \psi}\ L\wedge$$

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$$\frac{\phi, \phi_1, \ldots, \phi_n \Rightarrow \psi \qquad \phi', \phi_1, \ldots, \phi_n \Rightarrow \psi}{\phi \vee \phi', \phi_1, \ldots, \phi_n \Rightarrow \psi} L\vee$$

More arguments obtained from our knowledge base ...

Trusting the sister we get:

$$c_0 : \text{📢}_{\text{sis}} n, \text{📢✅}_{\text{sis}} n \Rightarrow n$$

$$c_1 : \text{📢}_{\text{sis}} n, \text{📢✅}_{\text{sis}} n \Rightarrow \text{💀}(\text{Nor})$$

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- If the sister is reliable , the brother is wrong !

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Analogously, trusting the brother we get:

$$d_0 : \text{📢}_{\text{bro}}a, \text{📢}^{\text{☑}}_{\text{bro}}a \Rightarrow \text{👁}_{\text{bro}}a$$

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Brief Detour: two ways to formally model defeasible inferences

Method 1. Use a deductive standard (e.g., classical logic) with "strict" inference rules and apply these to defeasible assumptions. (This part of the tutorial; but also Makinson's Plausible Assumptions [12], logic programming with default negation, adaptive logics, etc.)

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Method 2 Use strict assumptions in combination with defeasible inference rules. (E.g., Reiter's default logic [15].). Options:

- Nonmonotonic conditionals 📢$_{\text{agent}}\phi \rightsquigarrow \phi$ in the object language, equipped with generic defeasible modus ponens for $\rightsquigarrow$; or
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Approaches can often be translated: e.g., ASP and default logic in [11, 14].

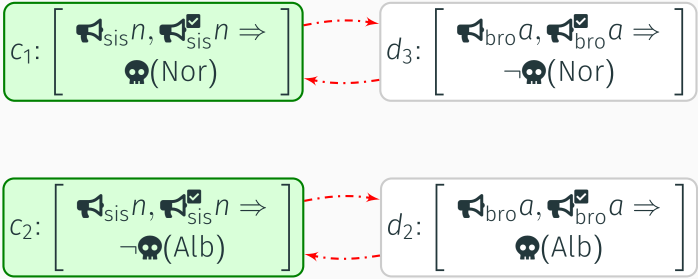
How to formally define argumentative attacks?

There are many options!

There are many options! You recall rebuttal?

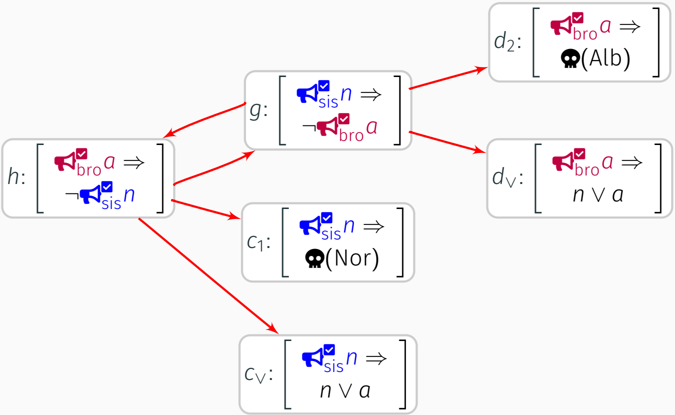
Definition

rebuttal. $\Gamma_1 \Rightarrow \phi$ rebuts $\Gamma_2 \Rightarrow -\phi$ if $\Gamma_2 \cap \mathcal{A}_d \neq \emptyset$ and where $-\phi = \psi$ if $\phi = \neg\psi$ and $-\phi = \neg\psi$ else.



Definition

direct defeat. $\Gamma_1 \Rightarrow \neg\phi$ directly defeats $\Gamma_2, \phi \Rightarrow \psi$, if $\phi \in \mathcal{A}_d$.



In the following we omit strict assumptions in the AFs, to avoid clutter.

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First we fix an argumentation semantics sem.

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How would you define a consequence relation for single extension semantics, such as grounded?

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How would you define a consequence relation for single extension semantics, such as grounded?

If the semantics leads to only one extension $\mathcal{E}$ (think: grounded), we can simply define:

- $\mathbb{K} \mid\!\sim \phi$ iff there is an argument a in $\mathcal{E}$ with conclusion ϕ.

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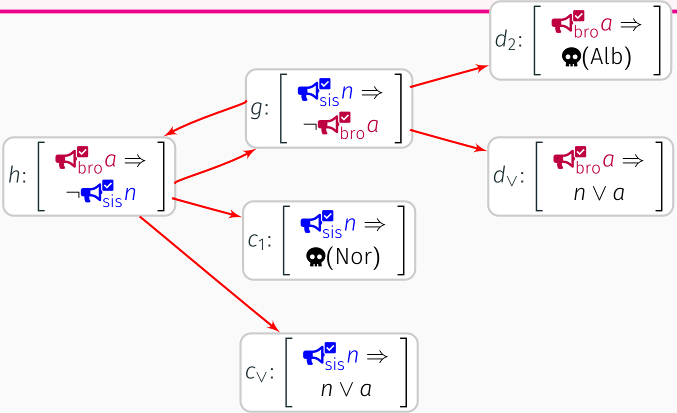
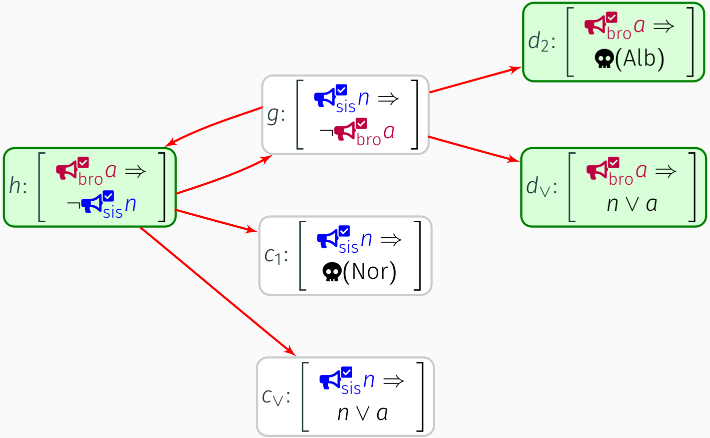
Problem

Many semantics have multiple extensions (think: stable semantics).

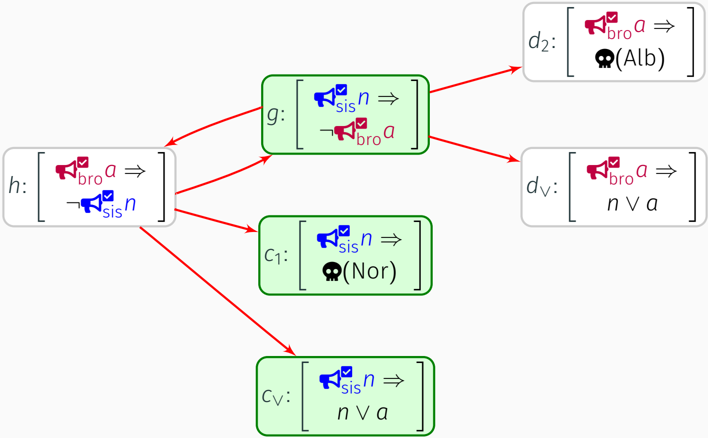
How many stable sets can you find?



Stable 1: the brother's right



Stable 2: the sister's right



So, how to define consequence for stable semantics?

Here's a natural idea:

- $\mathbb{K} \mid\!\sim \phi$ iff in every stable set $\mathcal{E}$ there is an argument a with conclusion ϕ.

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What do you think we get?

(Recall: $n = \text{💀}(\text{Nor}) \wedge \neg \text{💀}(\text{Alb})$ and $a = \text{💀}(\text{Alb}) \wedge \neg \text{💀}(\text{Nor})$))

- ☐ $\mathbb{K} \mid\!\sim n$
- ☐ $\mathbb{K} \mid\!\sim a$
- ☐ $\mathbb{K} \mid\!\sim n \vee a$

Here's a natural idea:

- $\mathbb{K} \mathrel{|\!\sim} \phi$ iff in every stable set $\mathcal{E}$ there is an argument a with conclusion ϕ.

What do you think we get?
(Recall: $n =$ 💀(Nor) $\wedge \neg$💀(Alb) and $a =$ 💀(Alb) $\wedge \neg$💀(Nor)))

- ☐ $\mathbb{K} \mathrel{|\!\sim} n$
- ☐ $\mathbb{K} \mathrel{|\!\sim} a$
- ☐ $\mathbb{K} \mathrel{|\!\sim} n \vee a$

The claim $n \vee a$ is a so-called floating conclusion. It is concluded by two otherwise conflicting arguments:

$c_\vee$: 📢$_{\text{sis}}n$, 📢☑$_{\text{sis}}n \Rightarrow n \vee a$

$d_\vee$: 📢$_{\text{bro}}a$, 📢☑$_{\text{bro}}a \Rightarrow n \vee a$

Do you have ideas of how else to define a consequence relation?

Definition

skeptical, shared arguments. $\mathbb{K} \mid\!\sim^{\mathsf{Att},\mathsf{sem}}_{\cap\mathsf{arg}} \phi$ iff there is an argument *a* with conclusions ϕ that is contained in every sem-extension of $\mathsf{AF}_{\mathsf{Att}}(\mathbb{K})$. "*intersection of arguments*"

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Note that with this consequence we don't get the floating conclusion $n \vee a$.

Can you see why?

check

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Note that with this consequence we don't get the floating conclusion $n \vee a$.

Can you see why?

check

We already had:

Definition

skeptical, shared conclusions. $\mathbb{K} \mid\!\sim^{\mathsf{Att},\mathsf{sem}}_{\cap\mathsf{con}} \phi$ iff in every sem-extension $\mathcal{S}$ of $\mathsf{AF}_{\mathsf{Att}}(\mathbb{K})$ there is an argument *a* with conclusion ϕ. *"intersection of consequences"*

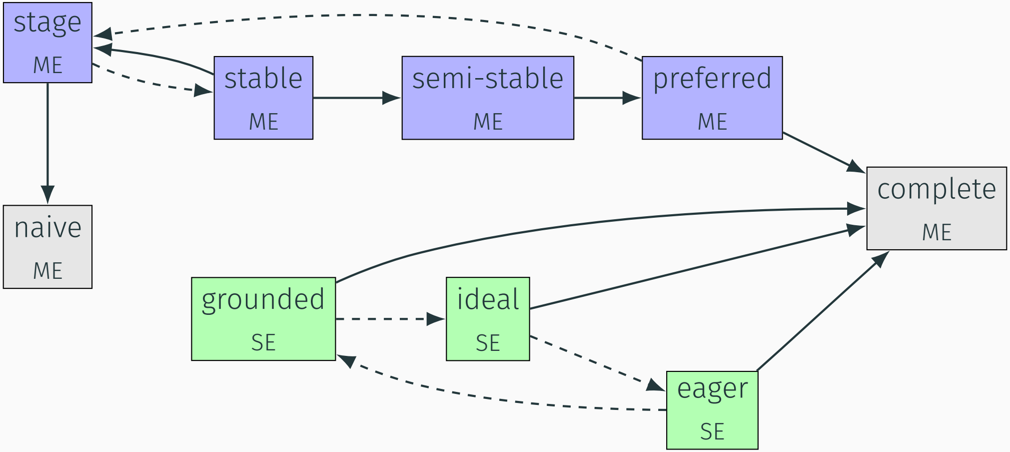
And there is:

Definition

credulous. $\mathbb{K} \mid\!\sim_{\cup}^{\text{Att,sem}} \phi$ iff there is a sem-extension in which there is an argument a with conclusion ϕ.

Part 4. Some Metatheory and some Subtleties

Semantic collapse



Solid arrows: general logical relations. Dashed arrows: additional logical relations in sequent-based argumentation where Att ∈ {{DirDef}, {DirDef, ConDef}, {Def}}. [2]

Caution

> ❗ When defining attacks and using semantics, one has to be careful.

In the following we highlight some dangers and then list some results about "good combinations".

THE KING was so proud of princess Charlotte. He saw her being a brave fighter.

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We get the new argument:

$$i : \text{👁}_{\text{king}}\text{brave}(\text{Cha}),\ \text{👁}^{\text{☑}}_{\text{king}}\text{brave}(\text{Cha}) \Rightarrow \text{brave}(\text{Cha})$$

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In order to highlight a problem we also note the following arguments:

$x_0 :$ 📢$_{\text{sis}}n$, 📢$^{✓}_{\text{sis}}n$, 📢$_{\text{bro}}a$, 📢$^{✓}_{\text{bro}}a$ $\Rightarrow \bot$

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$$x_0: \;\; 📢_{\text{sis}}n,\, 📢^{☑}_{\text{sis}}n,\, 📢_{\text{bro}}a,\, 📢^{☑}_{\text{bro}}a \;\Rightarrow\; \bot$$

$$x_1: \;\; 📢_{\text{sis}}n,\, 📢^{☑}_{\text{sis}}n,\, 📢_{\text{bro}}a,\, 📢^{☑}_{\text{bro}}a \;\Rightarrow\; \neg 👁^{☑}_{\text{king}}\text{brave}(\text{Cha})$$

THE KING was so proud of princess Charlotte. He saw her being a brave fighter.

We get the new argument:

$$i: \ \text{👁}_{\text{king}}\text{brave}(\text{Cha}), \ \text{👁}^{☑}_{\text{king}}\text{brave}(\text{Cha}) \ \Rightarrow \ \text{brave}(\text{Cha})$$

In order to highlight a problem we also note the following arguments:

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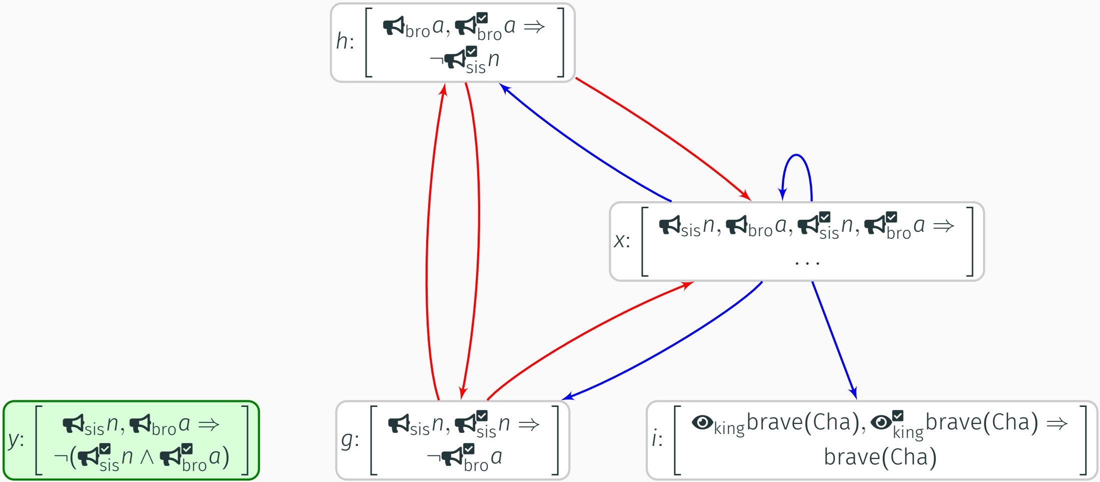
$$x_1: \ \text{📢}_{\text{sis}}n, \text{📢}^{☑}_{\text{sis}}n, \text{📢}_{\text{bro}}a, \text{📢}^{☑}_{\text{bro}}a \ \Rightarrow \ \neg\text{👁}^{☑}_{\text{king}}\text{brave}(\text{Cha})$$

$$y: \ \text{📢}_{\text{sis}}n, \text{📢}_{\text{bro}}a \ \Rightarrow \ \neg(\text{📢}^{☑}_{\text{sis}}n \wedge \text{📢}^{☑}_{\text{bro}}a)$$



Problem

Inconsistent arguments contaminate the grounded selection.

Where $\mathrm{sem}(\mathbb{K}, \mathrm{Att})$ denotes the set of all sem-extensions of the AF based on $\mathbb{K}$ and Att.

Definition

non-interference [19, 9]. Let $\mathbb{K}$ be a knowledge base. Where Γ is syntactically unrelated to $\mathbb{K}$ and consistent with $\mathcal{A}_s$. Let $\mathbb{K} \oplus \Gamma = \langle \mathcal{A}_s \cup \Gamma, \mathcal{A}_d \rangle$,

$$\mathrm{sem}(\mathbb{K}, \mathrm{Att}) = \{\mathcal{X} \cap \mathrm{Arg}(\mathbb{K}) \mid \mathcal{X} \in \mathrm{sem}(\mathbb{K} \oplus \Gamma, \mathrm{Att})\}.$$

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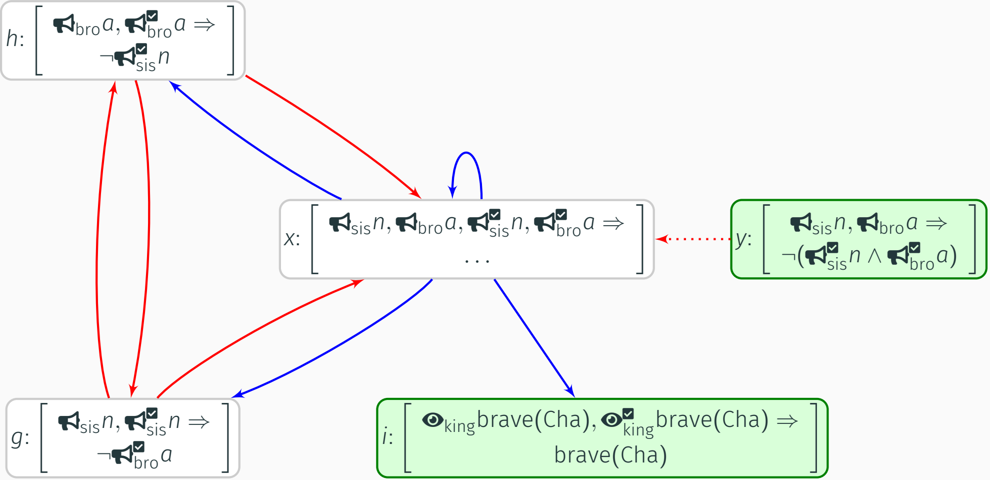
Proposition 2

Non-interference is, in general, not satisfied for $\mathrm{Att} = \{$direct defeat$\}$ and grounded semantics.

But how to achieve this. How to block arguments that contaminate the argumentation framework (due to logical explosion)?

Definition

consistency defeats. $\Gamma_1 \Rightarrow \neg \bigwedge \Gamma_2$ consistency defeats $\Gamma_2, \Gamma_2' \Rightarrow \psi$, if $\Gamma_1 \subseteq \mathcal{A}_s$ and $\Gamma_2 \subseteq \mathcal{A}_d$.



Results: Non-interference

Proposition 3

Let sem $\in$ {stable, grounded}. Non-interference is satisfied for Att $\in$ {{direct defeat, consistency defeat}, {defeat}} and sem-semantics.

where

- $\Gamma \Rightarrow \neg \bigwedge \Delta$ defeats $\Lambda \Rightarrow \phi$ iff $\emptyset \neq \Delta \subseteq \Lambda$

So far, we have considered a binary conflict: the one between what the sister and what the brother states. Triple conflicts are conflicts between three statements. They come with their own problems ...

A Story with a Triple conflict

NORBERT AND ALBERT are the last dragons. They are twins, only distinguished by the fact that Norbert spouts blue fire, while Albert spouts red fire.

This gives rise to the following knowledge base:

$$\mathcal{A}_S = \{\text{👁}_{\text{sis}}\text{💀}(\text{Nor}),\ \text{👁}_{\text{bro}}\text{💀}(\text{Alb}),\ \text{👁}_{\text{king}}\neg(\text{💀}(\text{Nor}) \wedge \text{💀}(\text{Alb}))\}$$

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A Story with a Triple conflict

NORBERT AND ALBERT are the last dragons. They are twins, only distinguished by the fact that Norbert spouts blue fire, while Albert spouts red fire. At the great battle against the underworld, the princess' brother saw her killing the red fire spouting Albert , and her sister saw her killing the blue fire spouting Norbert .

This gives rise to the following knowledge base:

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A Story with a Triple conflict

NORBERT AND ALBERT are the last dragons. They are twins, only distinguished by the fact that Norbert spouts blue fire, while Albert spouts red fire. At the great battle against the underworld, the princess' brother saw her killing the red fire spouting Albert , and her sister saw her killing the blue fire spouting Norbert . However, the king saw a dragon flying over the castle right after the battle .

This gives rise to the following knowledge base:

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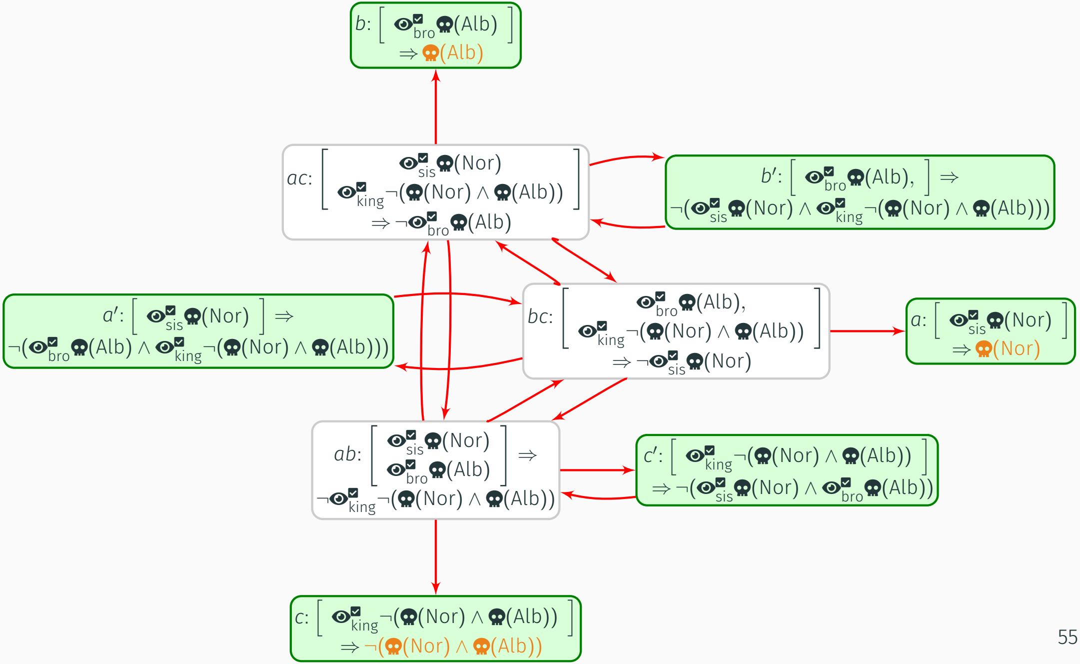

b: [bro (Alb)] ⇒ (Alb)
ac: [sis (Nor) king ¬((Nor) ∧ (Alb))] ⇒ ¬ bro (Alb)
b′: [bro (Alb),] ⇒ ¬(sis (Nor) ∧ king ¬((Nor) ∧ (Alb)))
a′: [sis (Nor)] ⇒ ¬(bro (Alb) ∧ king ¬((Nor) ∧ (Alb)))
bc: [bro (Alb), king ¬((Nor) ∧ (Alb))] ⇒ ¬ sis (Nor)
a: [sis (Nor)] ⇒ (Nor)
ab: [sis (Nor) bro (Alb)] ⇒ ¬ king ¬((Nor) ∧ (Alb))
c′: [king ¬((Nor) ∧ (Alb))] ⇒ ¬(sis (Nor) ∧ bro (Alb))
c: [king ¬((Nor) ∧ (Alb))] ⇒ ¬((Nor) ∧ (Alb))

Rationality postulates

Let $\mathcal{E} \in \text{sem}(\mathbb{K}, \text{Att})$.

Definition

consistency of extensions. The set $\{\phi \mid \Gamma \Rightarrow \phi \in \mathcal{E}\}$ is consistent.

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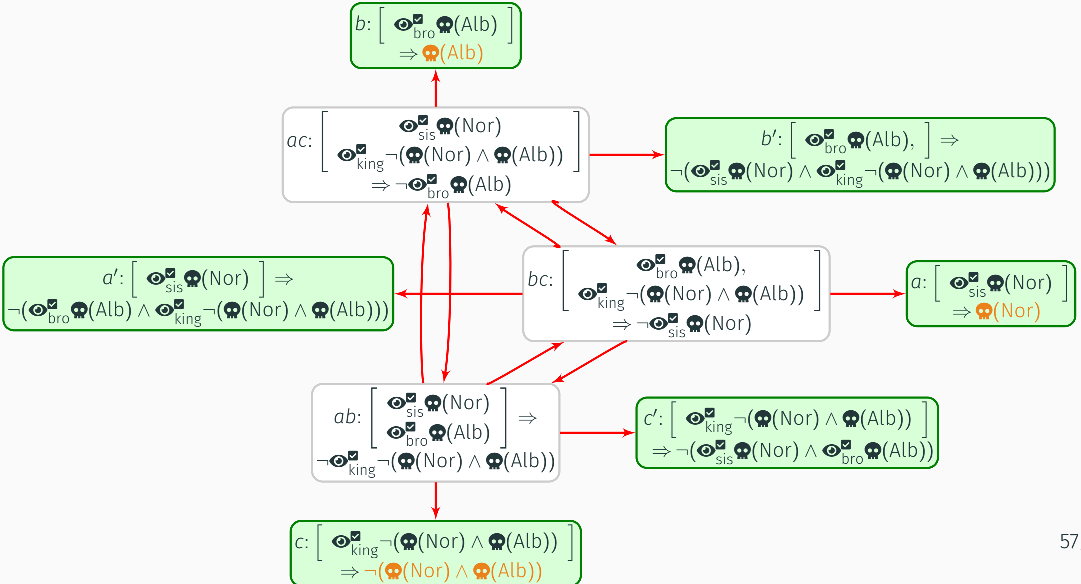
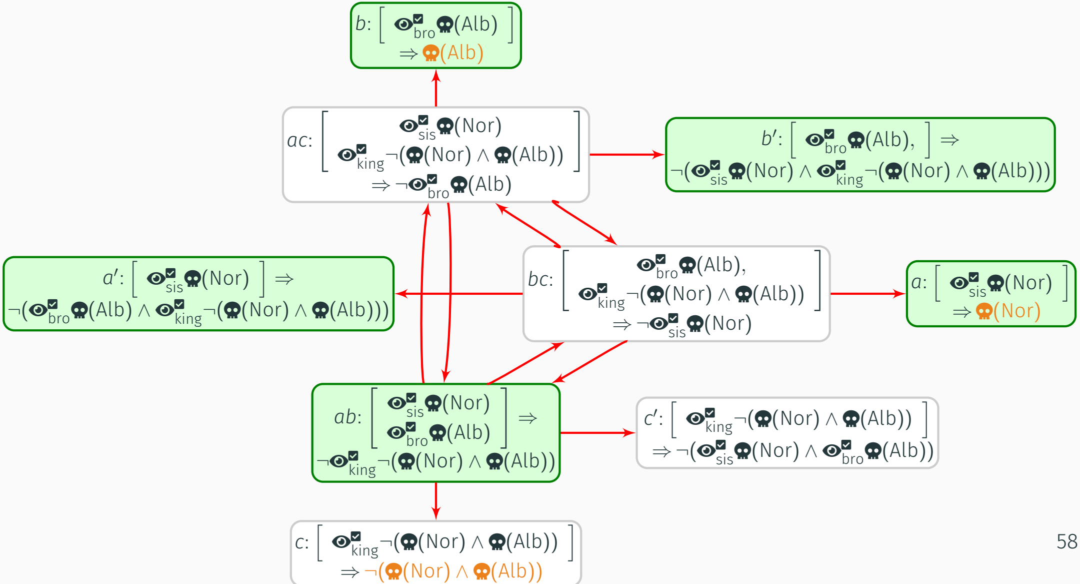
Proposition 5

With $\text{Att} = \{\text{Defeat}\}$ and $\text{sem} = \text{stable}$, consistency of extension is violated.

The same scenario with direct defeat ...



The same scenario with direct defeat … a stable extension



Let $\mathcal{E} \in \mathsf{sem}(\mathbb{K}, \mathsf{Att})$.

Definition

logical closure. For all $\psi \in \mathsf{Cn}(\{\phi \mid \Gamma \Rightarrow \phi \in \mathcal{E}\})$ there is a $\Gamma \Rightarrow \psi \in \mathcal{E}$.

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Theorem 7

Let $\mathbb{K}$ be a knowledge base.

1. Stable semantics satisfies non-interference, consistency and logical closure for $\mathsf{Att} \in \{\{\mathsf{DirectDefeat}, \mathsf{ConsistencyDefeat}\}, \{\mathsf{DirectDefeat}\}\}$.
2. Grounded semantics satisfies non-interference, consistency and logical closure for $\mathsf{Att} \in \{\{\mathsf{DirectDefeat}, \mathsf{ConsistencyDefeat}\}, \{\mathsf{Defeat}\}\}$.

Let $\mathcal{E} \in \mathsf{sem}(\mathbb{K}, \mathsf{Att})$.

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Theorem 8

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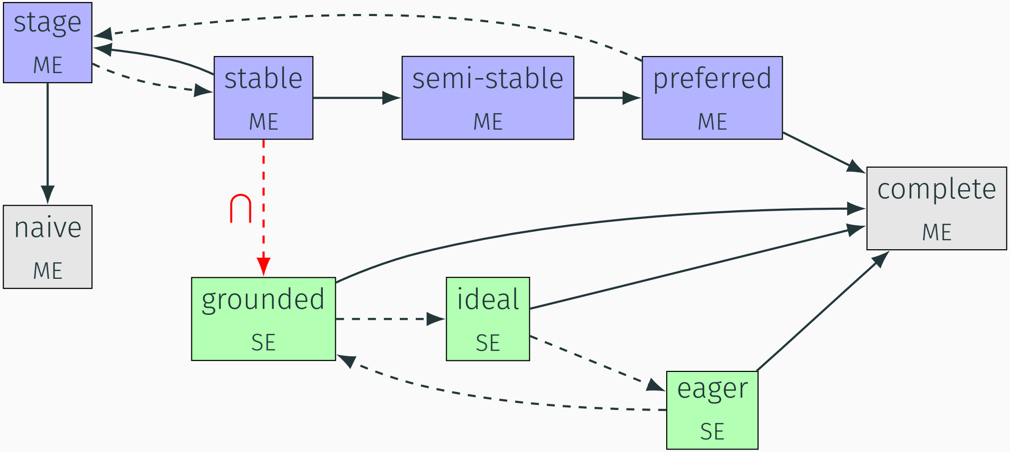
2. Grounded semantics satisfies non-interference, consistency and logical closure for $\mathsf{Att} \in \{\{\mathsf{DirectDefeat}, \mathsf{ConsistencyDefeat}\}, \{\mathsf{Defeat}\}\}$.

!

So, we found some good combos! :-)

Semantic collapse



Solid arrows: general logical relations. Dashed arrows: additional logical relations in sequent-based argumentation where Att = {DirDef, ConDef}. [2]

It speaks in favor of a given KRR-method's transparency if it coheres with perspectives provided by other KRR-methods.

MCS representation

It speaks in favor of a given KRR-method's transparency if it coheres with perspectives provided by other KRR-methods.

How does our argumentation-based method fair in this respect?

Definition

Let $\mathbb{K} = \langle \mathcal{A}_s, \mathcal{A}_d \rangle$ be a knowledge base and $\mathcal{A} \subseteq \mathcal{A}_d$.
$\mathcal{A}$ is maximally consistent in $\mathbb{K}$ iff

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We let maxcon($\mathbb{K}$) be the set of all maximal consistent set of defeasible assumptions in $\mathbb{K}$.

Let us go back to our knowledge base $\mathbb{K} = \langle \mathcal{A}_s, \mathcal{A}_d \rangle$ with

$$\mathcal{A}_s = \{\text{👁}_{\text{sis}}\text{💀}(\text{Nor}),\ \text{👁}_{\text{bro}}\text{💀}(\text{Alb}),\ \text{👁}_{\text{king}}\neg(\text{💀}(\text{Nor}) \vee \text{💀}(\text{Alb}))\}$$

$$\mathcal{A}_d = \{\text{👁}^{\text{☑}}_{\text{agent}}\phi \mid \phi \in \mathcal{L}\}$$

What are the maximally consistent sets for $\mathbb{K}$? How many are there?

maxcon($\mathbb{K}$) consists of:

$$\mathcal{M}_1 = \mathcal{A}_d \setminus \{\text{👁}^{\text{☑}}_{\text{king}}\neg(\text{💀}(\text{Nor}) \wedge \text{💀}(\text{Alb}))\},$$
$$\mathcal{M}_2 = \mathcal{A}_d \setminus \{\text{👁}^{\text{☑}}_{\text{sis}}\text{💀}(\text{Nor})\},$$
$$\text{and } \mathcal{M}_3 = \mathcal{A}_d \setminus \{\text{👁}^{\text{☑}}_{\text{bro}}\text{💀}(\text{Alb})\}$$

Let us compare this to our stable extensions when working with direct defeat. AF maxcon($\mathbb{K}$) consists of:

$$\mathcal{M}_1 = \mathcal{A}_d \setminus \{👁^{☑}_{\text{king}} \neg(💀(\text{Nor}) \wedge 💀(\text{Alb}))\},$$
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$$\text{and } \mathcal{M}_3 = \mathcal{A}_d \setminus \{👁^{☑}_{\text{bro}} 💀(\text{Alb})\}$$

Let us compare this to our stable extensions when working with direct defeat. AF

Hm, ... it looks suspiciously as if the stable sets coincide with the maxicon sets!

And indeed ... when working with direct defeats we have:

Theorem 9

Let $\mathbb{K}$ be a knowledge base and $\mathsf{Att} \in \{\{\text{direct defeat}\}, \{\text{direct defeat}, \text{consistency defeat}\}\}$. Then,

$$\mathsf{stable}(\mathbb{K}, \mathsf{Att}) = \{\mathsf{Arg}(\mathcal{M}) \mid \mathcal{M} \in \mathsf{maxcon}(\mathbb{K})\}.$$

And indeed ... when working with direct defeats we have:

Theorem 10

Let $\mathbb{K}$ be a knowledge base and $\mathsf{Att} \in \{\{\text{direct defeat}\}, \{\text{direct defeat}, \text{consistency defeat}\}\}$. Then,

$$\mathsf{stable}(\mathbb{K}, \mathsf{Att}) = \{\mathsf{Arg}(\mathcal{M}) \mid \mathcal{M} \in \mathsf{maxcon}(\mathbb{K})\}.$$

Similar results relative to variants of default logic and constrained input/output logic can be obtained when incorporating default rules in the knowledge base! (See talk next week!)

Small exercise

Suppose our knowledge base is $\mathbb{K}_1 = \langle \mathcal{A}_s, \mathcal{A}_d \rangle$ with

- $\mathcal{A}_s = \emptyset$ and
- $\mathcal{A}_d = \{p \wedge q, \neg p \wedge q, s\}$

Small exercise

Suppose our knowledge base is $\mathbb{K}_1 = \langle \mathcal{A}_s, \mathcal{A}_d \rangle$ with

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1. What are the maximal consistent subsets of $\mathbb{K}$?

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2. How do the stable extensions look like for $\mathsf{Att} = \{\text{direct defeat}\}$?

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1. The maximal consistent subsets are $\mathcal{M}_1 = \{p \wedge q, s\}$ and $\mathcal{M}_2 = \{\neg p \wedge q, s\}$.
2. The stable extensions are Arg($\mathcal{M}_1$) and Arg($\mathcal{M}_2$).

Can we find something similar for grounded semantics?

Theorem 11

Let $\mathbb{K}$ be a knowledge base, $\mathsf{free}(\mathbb{K}) = \bigcap \mathsf{maxcon}(\mathbb{K})$ and $\mathsf{Att} \in \{\{\text{direct defeat}, \text{consistency defeat}\}, \{\text{defeat}\}\}$.
Then,

$$\mathsf{grounded}(\mathbb{K}, \mathsf{Att}) = \{\mathsf{Arg}(\mathsf{free}(\mathbb{K}))\}.$$

Back to our little exercise …

Suppose our knowledge base is $\mathbb{K}_1 = \langle \mathcal{A}_s, \mathcal{A}_d \rangle$ with

- $\mathcal{A}_s = \emptyset$ and
- $\mathcal{A}_d = \{p \wedge q, \neg p \wedge q, s\}$

1. What is the set $\text{free}(\mathbb{K}_1)$?

Suppose our knowledge base is $\mathbb{K}_1 = \langle \mathcal{A}_s, \mathcal{A}_d \rangle$ with

- $\mathcal{A}_s = \emptyset$ and
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1. What is the set free($\mathbb{K}_1$)?
2. What is the grounded extension for Att$\{\{$direct defeat, consistency defeat$\}, \{$defeat$\}\}$?

Suppose our knowledge base is $\mathbb{K}_1 = \langle \mathcal{A}_s, \mathcal{A}_d \rangle$ with

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1. What is the set $\text{free}(\mathbb{K}_1)$?
2. What is the grounded extension for $\text{Att}\big\{\{\text{direct defeat, consistency defeat}\}, \{\text{defeat}\}\big\}$?
3. What would change for $\mathbb{K}_2 = \langle \emptyset, \{p, q, \neg p, s\} \rangle$?

Suppose our knowledge base is $\mathbb{K}_1 = \langle \mathcal{A}_s, \mathcal{A}_d \rangle$ with

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Suppose our knowledge base is $\mathbb{K}_1 = \langle \mathcal{A}_s, \mathcal{A}_d \rangle$ with

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!

1. free($\mathbb{K}$) $= \{s\}$
2. grounded($\mathbb{K}$, Att) $= \{\mathrm{Arg}(\{s\})\}$.

Suppose our knowledge base is $\mathbb{K}_1 = \langle \mathcal{A}_s, \mathcal{A}_d \rangle$ with

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!

1. free($\mathbb{K}$) $= \{s\}$
2. grounded($\mathbb{K}$, Att) $= \{\text{Arg}(\{s\})\}$.
3. free($\mathbb{K}_2$) $= \{s, g\}$ and grounded($\mathbb{K}$, Att) $= \{\text{Arg}(\{q, s\})\}$.

What about consequence relations? Are there similar representational results?

Definition

skept., shared arguments. $\mathbb{K} \mathrel{|\!\sim}^{\mathsf{Att},\mathsf{sem}}_{\cap\mathsf{arg}} \phi$ iff $\exists \Gamma \Rightarrow \phi \in \bigcap \mathsf{sem}(\mathbb{K}, \mathsf{Att})$.

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Theorem 13

1. Where $\mathsf{sem} \in \{\mathsf{stable}\}$ and $\mathsf{Att} \in \{\mathsf{AttDir}, \mathsf{AttDirCon}\}$:

$$\mathbb{K} \mid\!\sim^{\mathsf{Att},\mathsf{sem}}_{\cap\mathsf{con}} \phi \quad \text{iff} \quad \phi \in \bigcap_{\mathcal{M} \in \mathsf{maxcon}(\mathbb{K})} \mathsf{Cn}(\mathcal{M} \cup \mathcal{A}_s)$$

Definition

skept., shared arguments. $\mathbb{K} \mid\!\sim^{\mathsf{Att},\mathsf{sem}}_{\cap\mathsf{arg}} \phi$ iff $\exists\Gamma \Rightarrow \phi \in \bigcap \mathsf{sem}(\mathbb{K}, \mathsf{Att})$.

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Theorem 14

1. Where sem $\in$ {stable} and Att $\in$ {AttDir, AttDirCon}:

$$\mathbb{K} \mid\!\sim^{\mathsf{Att},\mathsf{sem}}_{\cap\mathsf{con}} \phi \quad \text{iff} \quad \phi \in \bigcap_{\mathcal{M} \in \mathsf{maxcon}(\mathbb{K})} \mathsf{Cn}(\mathcal{M} \cup \mathcal{A}_{\mathsf{s}})$$

2. Where sem $\in$ {stable} and Att $\in$ {AttDir, AttDirCon}:

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Theorem 15

1. Where $\mathsf{sem} \in \{\mathsf{stable}\}$ and $\mathsf{Att} \in \{\mathsf{AttDir}, \mathsf{AttDirCon}\}$:

$$\mathbb{K} \mid\!\sim^{\mathsf{Att},\mathsf{sem}}_{\cap\mathsf{con}} \phi \quad \text{iff} \quad \phi \in \bigcap_{\mathcal{M}\in\mathsf{maxcon}(\mathbb{K})} \mathsf{Cn}(\mathcal{M} \cup \mathcal{A}_s)$$

2. Where $\mathsf{sem} \in \{\mathsf{stable}\}$ and $\mathsf{Att} \in \{\mathsf{AttDir}, \mathsf{AttDirCon}\}$:

$$\mathbb{K} \mid\!\sim^{\mathsf{Att},\mathsf{sem}}_{\cap\mathsf{arg}} \phi \quad \text{iff} \quad \phi \in \mathsf{Cn}(\bigcap \mathsf{maxcon}(\mathbb{K}) \cup \mathcal{A}_s)$$

3. Where $\star \in \{\cap\mathsf{con}, \cap\mathsf{arg}\}$ and $\mathsf{Att} \in \{\mathsf{AttDef}, \mathsf{AttDirCon}\}$:

$$\mathbb{K} \mid\!\sim^{\mathsf{Att},\mathsf{grounded}}_{\star} \phi \quad \text{iff} \quad \phi \in \mathsf{Cn}(\bigcap \mathsf{maxcon}(\mathbb{K}) \cup \mathcal{A}_s).$$

Cumulativity

Nonmonotonic consequence relations are non-robust: adding information to a knowledge base may destroy consequences.

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Can we identify some type of information that, if added to the knowledge base, does not lead to the loss of consequences? Ideas?

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Notation: let $\mathbb{K} \oplus \phi \in \{\langle \mathcal{A}_s \cup \{\phi\}, \mathcal{A}_s \rangle, \langle \mathcal{A}_s, \mathcal{A}_d \cup \{\phi\} \rangle\}$.

Definition

cautious monotony. $\mid\!\sim$ is cautious monotonic if the following holds: if

$$\mathbb{K} \mid\!\sim \phi \text{ and } \mathbb{K} \mid\!\sim \psi, \text{ then } \mathbb{K} \oplus \phi \mid\!\sim \psi.$$

Let $\mathbb{K} \oplus \phi \in \{\langle \mathcal{A}_s \cup \{\phi\}, \mathcal{A}_s\rangle, \langle \mathcal{A}_s, \mathcal{A}_d \cup \{\phi\}\rangle\}$.

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cautious monotony. $\mid\!\sim$ is cautious monotonic if the following holds:

whenever $\mathbb{K} \mid\!\sim \phi$ and $\mathbb{K} \mid\!\sim \psi$ then $\mathbb{K} \oplus \phi \mid\!\sim \psi$.

A similar property is

Definition

cautious cut. $\mid\!\sim$ satisfies cautious cut if the following holds: if $\mathbb{K} \mid\!\sim \phi$ and

$\mathbb{K} \oplus \phi \mid\!\sim \psi$, then $\mathbb{K} \mid\!\sim \psi$.

Let $\mathbb{K} \oplus \phi \in \{\langle \mathcal{A}_s \cup \{\phi\}, \mathcal{A}_s \rangle, \langle \mathcal{A}_s, \mathcal{A}_d \cup \{\phi\} \rangle\}$.

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cautious monotony. $\mid\!\sim$ is cautious monotonic if the following holds: whenever $\mathbb{K} \mid\!\sim \phi$ and $\mathbb{K} \mid\!\sim \psi$ then $\mathbb{K} \oplus \phi \mid\!\sim \psi$.

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cautious cut. $\mid\!\sim$ satisfies cautious cut if the following holds: if $\mathbb{K} \mid\!\sim \phi$ and $\mathbb{K} \oplus \phi \mid\!\sim \psi$, then $\mathbb{K} \mid\!\sim \psi$.

Putting things together

Definition

cumulativity. $\mid\!\sim$ is cumulative iff it satisfies cautious monotony and cautious cut.

Let $\mathbb{K} \oplus \phi \in \{\langle \mathcal{A}_s \cup \{\phi\}, \mathcal{A}_s \rangle, \langle \mathcal{A}_s, \mathcal{A}_d \cup \{\phi\} \rangle\}$.

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cumulativity. $\mid\!\sim$ is cumulative iff it satisfies cautious monotony and

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Result

Theorem 16

All consequence relations from Theorem 15 are cumulative.

Result

Theorem 17

All consequence relations from Theorem 15 are cumulative.

Disjunctive Reasoning

Definition

(OR). If $\mathbb{K} \oplus \gamma \mid\!\sim \phi$ and $\mathbb{K} \oplus \delta \mid\!\sim \phi$ then $\mathbb{K} \oplus (\gamma \vee \delta) \mid\!\sim \phi$.

Disjunctive Reasoning

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Consider $\mathbb{K} = \langle \mathcal{A}_s : \emptyset, \mathcal{A}_d \rangle$ with $\mathcal{A}_d = \{\neg p, \neg p \supset r, \neg q, \neg q \supset r\}$.

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- What are the maxicon sets when adding p to the strict (or defeasible) assumptions?
- Does r follow via $\mathrel{|\!\sim}^{\text{stable}}_{\cap\text{arg}}$?

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- Does r follow via $\mathrel{|}\!\sim^{\text{stable}}_{\cap\text{arg}}$?
- Does r follow when we add q to the strict (or defeasible) assumptions?
- What are the maxicon sets when adding $p \vee q$ to the strict (or defeasible) assumptions?
- Does r follow via $\mathrel{|}\!\sim^{\text{stable}}_{\cap\text{arg}}$ under this addition?

Disjunctive reasoning under shared conclusions

Theorem 18

$\mid\!\sim^{\mathsf{stable}}_{\cap\mathsf{con}}$ satisfies OR where $\mathsf{Att} \in \{\{\mathsf{DirDef}\}, \{\mathsf{DirDef}, \mathsf{ConDef}\}\}$.

Going beyond classical logic?

The presented results generalize to base logics that satisfy ([2]):[2]

- Simple 'reflexive' arguments: $\phi \Rightarrow \phi$ is an argument

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- Conjunction has its usual meaning:

$$\frac{\Gamma, \phi, \psi \Rightarrow \Delta}{\Gamma, \phi \wedge \psi \Rightarrow \Delta} \qquad \frac{\Gamma_1 \Rightarrow \Pi_1, \phi \quad \Gamma_2 \Rightarrow \Pi_2, \psi}{\Gamma_1, \Gamma_2 \Rightarrow \Pi_1, \Pi_2, \phi \wedge \psi}$$

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- Simple 'reflexive' arguments: $\phi \Rightarrow \phi$ is an argument
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- Argument construction is monotonic (attack take care of defeat):

Other Logics (WiP): Bochvar

- Let Γ be consistent. Then $\Gamma \vdash_{B3} \phi$ iff $\Gamma \vdash_{CL} \phi$ and Atoms(ϕ) $\subseteq$ Atoms(Γ).
- So:
  - $p \wedge q \vdash_{B3} p$
  - $p \vee q, \neg p \vdash_{B3} q$
  - $p \nvdash_{B3} p \vee q$
  - The logic 'stays on topic'.

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  - $p \nvdash_{B3} p \vee q$
  - The logic 'stays on topic'.

Definition

Reductio Attacks are defined as follows: $(\Gamma, \neg\phi)$ directly reductio-attacks $(\Gamma' \cup \{\phi'\}, \psi)$ if $(\phi', \phi) \in \text{Arg}$.

Translation

1. Where $\mathcal{E} \subseteq \text{Arg}_{\textbf{CL}}(\mathbb{K})$ let $\mathcal{E}^{\downarrow} = \mathcal{E} \cap \text{Arg}_{\textbf{B3}}(\mathbb{K})$.
2. Where $\mathcal{E} \subseteq \text{Arg}_{\textbf{B3}}(\mathbb{K})$ let $\mathcal{E}^{\uparrow} = \{(\Gamma, \phi) \mid (\Gamma, \phi') \in \mathcal{E} \text{ and } \phi' \vdash_{\textbf{CL}} \phi\}$.

Theorem 19

Let *AF* and *AF′* be based on the knowledge base $\mathbb{K}$. In case of *AF* the underlying logic is **CL** and the underlying attack is direct defeat, while in the case of *AF′* it is **B3** and direct reductio. Then,

1. For each $\mathcal{E} \in \text{stable}(AF)$, $\mathcal{E}^{\uparrow} \in \text{stable}(AF')$.
2. For each $\mathcal{E} \in \text{stable}(AF')$, $\mathcal{E}^{\downarrow} \in \text{stable}(AF)$.

So: $\mathcal{S} \mid\!\sim^{\cap\text{stable}}_{\text{CL}} \phi$ iff $\phi \in \text{Cn}_{\text{CL}}(\{\psi \mid \mathcal{S} \mid\!\sim^{\text{stable}}_{\text{B}} \psi\})$.

References

Here we only scratched the surface of the meta-theory of sequent-based argumentation. For a rather deep dive into this topic check out the recent:

- Arieli, Ofer, Borg, AnneMarie, & Straßer, Christian (2023). A postulate-deriven study of logical argumentation. Artificial Intelligence. [2]
- Arieli, O., & Christian Straßer (2015). Sequent-Based Logical Argumentation. Argument and Computation., 6(1), 73–99. [4]

For a more general overview on logical argumentation and its meta-theory see:

- Arieli, O., Borg, A., Heyninck, J., & Straßer, C. (2021). Logic-based approaches to formal argumentation. Journal of Applied Logics-IfCoLog Journal, 8(6), 1793–1898. [6]

More references

Applications of sequent-based argumention:

- normative reasoning [16, 8]
- explanations [5]
- automated proofs [3, 1]
- probabilistic reasoning, cognition [17]

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Part 5: Application to normative reasoning

Part 5: Normative reasoning

Normative reasoning:

- Drawing conclusions from and about obligations, prohibitions, permissions, rights, violations...
- Important to law, ethics, AI, business protocols, social interaction...

Norms influence everyday decision-making and the way (AI) agents shape their world.

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But what if we then learn that “dragons ought to be left in peace”?

Normative reasoning is **highly defeasible** too! !

Part 5: Normative reasoning

Some more examples:

Violation reasoning (business policy):

- Private information must not be disclosed. If nevertheless disclosed, a correction procedure must be initiated.

Exception reasoning (traffic law):

- You ought to drive on the right. When you overtake another vehicle you ought to drive on the left.

Dilemmas (medical ethics):

- Should an organ be given to 90-year old who is first on a donor waiting list, or to a teenager who needs it now?

Conflict resolution mechanisms have been developed for such scenarios.

Part 5: Deontic logic

Deontic logic: formal field that deals with normative reasoning.[1]

- Around since the 1950s (von Wright)
- **Traditionally**: monotonic modal logics:

  $\mathcal{O}$ is a modality for 'It ought to be that'

  '$\mathcal{O}$(promise)' for 'It ought to be that Charlotte keeps her promise'

Developments in computer science led to nonmonotonic deontic logics:

- Often non-modal logic.
- **Input/Output logic** (Makinson and van der Torre, 2001) This tutorial!

[1] Greek word déon refers to 'that which is binding': duty.

Problem: Deontic logics show that an obligation holds, but **don't show how** conflicts are addressed.

Main goals of Part 5 (based on van Berkel and Strasser, 2022):

1. Adopt a **proof calculus** that generates deontic (counter-)arguments
2. Use of **formal argumentation** to **transparently** characterize conflict resolution in defeasible normative reasoning.

Argumentation serves **explainability** due to its closeness to human reasoning (Mercier and Sperber, 2011).

→ see also ArgXAI on Monday!

The formal language:

1. Using **labelled versions** of a propositional language:

   ϕ^f = 'ϕ is a fact.'

   ϕ^o = 'ϕ is obligatory.'

   ϕ^c = 'ϕ is constraint with which obligations must be consistent.'

   The language makes transparent the various roles formulas play:

   $$\texttt{prom}^f \text{ vs } \texttt{prom}^o$$

2. Adopting **norms as objects of reasoning**:

   $(\phi, \psi)^n$ and $\neg(\phi, \psi)^n$

   e.g., $(\texttt{prom}, \texttt{hunt})^n$ = 'If Charlotte promised to, she ought to hunt Albert.'

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   ϕ^c = 'ϕ is constraint with which obligations must be consistent.'

   The language makes transparent the various roles formulas play:

   $$\texttt{prom}^f \text{ vs } \texttt{prom}^o$$

2. Adopting **norms as objects of reasoning**:

   $(\phi, \psi)^n$ and $\neg(\phi, \psi)^n$

   e.g., $(\texttt{prom}, \texttt{hunt})^n$ = 'If Charlotte promised to, she ought to hunt Albert.'

   e.g., $\neg(\texttt{prom}, \texttt{hunt})^n$ = 'the norm $(\texttt{prom}, \texttt{hunt})^n$ inapplicable.'

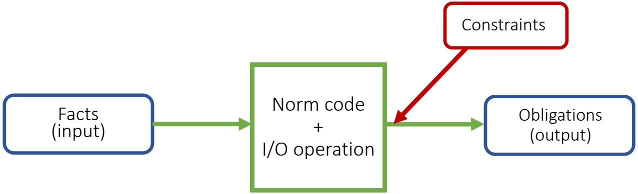
Part 5: Proof calculus

A Normative Knowledge Base $\mathbb{K} = \langle \mathcal{F}, \mathcal{N}, \mathcal{C} \rangle$:

- $\mathcal{F}$ is the factual context.
- $\mathcal{N}$ is a normative code.
- $\mathcal{C}$ are constraints with which inferred obligations must be consistent.

Obligations are not part of the knowledge base: **they are derived!**

The basic idea (in the spirit of Input/Output Logic):



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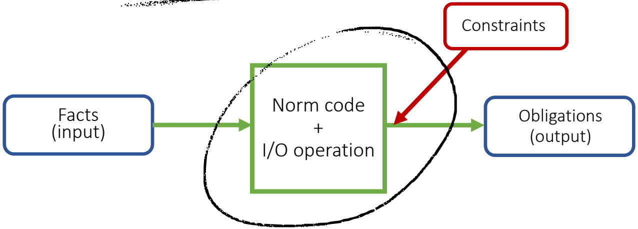

We are interested in generating **two types of arguments:**

1. Giving **reasons for obligations**:

   e.g., $\mathtt{prom}^f, (\mathtt{prom}, \mathtt{hunt})^n \Rightarrow \mathtt{hunt}^o$.

2. Giving **reasons for norm inapplicability** (attackers!):

   e.g. $\mathtt{prom}^f, \neg\mathtt{hunt}^c \Rightarrow \neg(\mathtt{prom}, \mathtt{hunt})^n$

How are such arguments derived? **The calculus!**

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How are such arguments derived? **The calculus!**

Proof systems (briefly):

- Concern derivability, not satisfiability and validity
- Axiomatic systems, **sequent calculi**, natural deduction, tableaux,...
- Proofs are formalized as mathematical objects in their own right

We focus on sequent systems (Gentzen, 1934):

- Rule-based proof systems (in contrast to axiomatic systems)
- Provides a constructive approach to studying properties of logics
- Useful for automated reasoning procedures

A Deontic Argumentation Calculus (DAC):

- Rule-based proof system for generating arguments (recall LK):

$$\Gamma \Rightarrow \Delta$$

A DAC-derivation of $\Gamma \Rightarrow \Delta$ is a tree-like structure:

1. whose leaves are initial sequents,
2. whose root is $\Gamma \Rightarrow \Delta$, and
3. whose rule applications are instances of the calculus' rules.

Let's look at the rules.

A Deontic Argumentation Calculus:

$$\frac{}{\Gamma^i \Rightarrow \Delta^i}\ Ax \text{, for } i \in \{f, o, c\} \text{ and } \Gamma^i \Rightarrow \Delta^i \text{ is LK derivable}$$

$$\frac{}{\phi^f, (\phi, \psi)^n \Rightarrow \psi^o}\ \textit{F-Detach}$$

$$\frac{\phi^f, \Gamma \Rightarrow \Delta}{\phi^o, \Gamma \Rightarrow \Delta}\ \textit{D-Detach} \qquad \frac{\Gamma \Rightarrow \phi^o}{\Gamma, (\neg\phi)^c \Rightarrow}\ \textit{Cons} \qquad \frac{\Gamma, (\phi, \psi) \Rightarrow}{\Gamma \Rightarrow \neg(\phi, \psi)^n}\ \textit{Inapp}$$

$$\frac{\Gamma \Rightarrow \phi \qquad \phi, \Gamma' \Rightarrow \Delta}{\Gamma, \Gamma' \Rightarrow \Delta}\ \textit{Cut}$$

Note: Just one calculus of many!

Ax Taking **labelled versions of any LK-derivable** arguments $\Gamma \Rightarrow \Delta$ as an initial sequents.

$$\frac{}{\Gamma^i \Rightarrow \Delta^i}\ Ax$$

Part 5: The rules

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$$\frac{}{\Gamma^i \Rightarrow \Delta^i}\,Ax$$

F-Detach Introducing **initial arguments**:

$$\frac{}{\phi^f, (\phi, \psi)^n \Rightarrow \psi^o}\,F\text{-}Detach$$

which detach obligations from facts and a norm.

Note! F-Detach is a Toulmin argument: premise, warrant, conclusion.

D-Detach The rule

$$\frac{\phi^f, \Delta \Rightarrow \Gamma}{\phi^o, \Delta \Rightarrow \Gamma}\ \textit{D-Detach}$$

captures **deontic detachment**:

- a norm may likewise be triggered by the output of some other norm.

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Part 5: The rules

Suppose we have the following norms: $(p, q \vee r)^n$, $(\top, \neg r)^n$, and $(q, z)^n$.

$$
\dfrac{
  \dfrac{
    \dfrac{}{p^f, (p, q \vee r)^n \Rightarrow (q \vee r)^o}\,\text{F-Det}
    \qquad
    \dfrac{
      \dfrac{}{\top^f, (\top, \neg r)^n \Rightarrow \neg r^o}\,\text{F-Det}
      \quad
      \dfrac{}{(q \vee r)^o, \neg r^o \Rightarrow q^o}\,\text{Ax}
    }{(q \vee r)^o, \top^f, (\top, \neg r)^n \Rightarrow q^o}\,\text{Cut}
  }{p^f, \top^f, (p, q \vee r)^n, (\top, \neg r)^n \Rightarrow q^o}\,\text{Cut}
  \qquad
  \dfrac{
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  }{q^o, (q, z)^n \Rightarrow z^o}\,\text{D-Det}
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$$

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$$
\begin{prooftree}
\AxiomC{}
\RightLabel{F-Det}
\UnaryInfC{$p^f, (p, q \vee r)^n \Rightarrow (q \vee r)^o$}
\AxiomC{}
\RightLabel{F-Det}
\UnaryInfC{$\top^f, (\top, \neg r)^n \Rightarrow \neg r^o$}
\AxiomC{}
\RightLabel{Ax}
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\RightLabel{Cut}
\BinaryInfC{$(q \vee r)^o, \top^f, (\top, \neg r)^n \Rightarrow q^o$}
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Cons Reasoning with **constraints**:[2]

$$\frac{\Delta \Rightarrow \phi^{o}}{\Delta, \neg\phi^{c} \Rightarrow}\ \textit{Cons} \qquad \frac{\texttt{prom}^{f}, (\texttt{prom}, \texttt{hunt})^{n} \Rightarrow \texttt{hunt}^{o}}{\texttt{prom}^{f}, (\texttt{prom}, \texttt{hunt})^{n}, \neg\texttt{hunt}^{c} \Rightarrow}\ \textit{Cons}$$

i.e. $\texttt{prom}^{f}$ and $(\texttt{prom}, \texttt{hunt})^{n}$ (reasons for $\texttt{hunt}^{o}$) are inconsistent with $\neg\texttt{hunt}^{c}$.

[2] nb. an empty right-hand side expresses inconsistent reasons.

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Inapp If the reasons are inconsistent, at least one **norm is inapplicable**!

$$\frac{\Delta, (\phi, \psi)^n \Rightarrow}{\Delta \Rightarrow \neg(\phi, \psi)^n} \textit{ Inapp} \qquad \frac{\texttt{prom}^f, (\texttt{prom}, \texttt{hunt})^n, \neg\texttt{hunt}^c \Rightarrow}{\texttt{prom}^f, \neg\texttt{hunt}^c \Rightarrow \neg(\texttt{prom}, \texttt{hunt})^n} \textit{ Inapp}$$

i.e. $\texttt{prom}^f$ and $\neg\texttt{hunt}^c$ are reasons for the inapplicability of $(\texttt{prom}, \texttt{hunt})^n$

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Part 5: The rules

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i.e. $\mathtt{prom}^f$ and $\neg\mathtt{hunt}^c$ are reasons for the inapplicability of $(\mathtt{prom}, \mathtt{hunt})^n$

Attacks all arguments using $(\mathtt{prom}, \mathtt{hunt})^n$ as a reason!

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Let's illustrate the use of DAC!

Deontic logic is driven by paradoxes and challenging scenarios.

Central challenge: **contrary-to-duty reasoning** (Chellas, 1963).

- Scenarios where an agent is bound by an initial duty, fails to comply, and a violation ensues.
- **Task:** agent must find out **what to do given her violation**.

Defeasible reasoning can adequately address CTD reasoning (whereas traditional deontic logics cannot).

Part 5: CTD scenario

> A ROYAL DECREE: HUNTING DRAGONS IS FORBIDDEN. However, if such a hunt would nevertheless take place, the hunter ought to ask the dragon for consent. Furthermore, to not frighten any dragons, if no hunt takes place, no consent should be asked either.

The normative knowledge base $\mathbb{K}_1$:

$$\mathcal{N} = \{(\top, \neg\mathtt{hunt})^n, (\mathtt{hunt}, \mathtt{cons})^n, (\neg\mathtt{hunt}, \neg\mathtt{cons})^n\}$$

$\mathcal{F} = \{\top^f\}$ (no specific facts given)

$\mathcal{C} = \{\top^c\}$ (no specific constraints given, only consistency)

We are only interested in $\mathbb{K}_1$ arguments $\Gamma \Rightarrow \Delta$, i.e., for which $\Gamma \subseteq \mathcal{N} \cup \mathcal{F} \cup \mathcal{C}$.

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Given the **compliant situation** $\mathcal{F} = \{\top^f\}$, we derive:

$$\dfrac{}{a: \quad \top^f, (\top, \neg\texttt{hunt})^n \Rightarrow \neg\texttt{hunt}^o}\ \textit{F-Detach}$$

and

$$\dfrac{a \qquad \dfrac{\dfrac{}{\neg\texttt{hunt}^f, (\neg\texttt{hunt}, \neg\texttt{cons})^n \Rightarrow \neg\texttt{cons}^o}\ \textit{F-Detach}}{\neg\texttt{hunt}^o, (\neg\texttt{hunt}, \neg\texttt{cons})^n \Rightarrow \neg\texttt{cons}^o}\ \textit{D-Detach}}{b: \quad \top^f, (\top, \neg\texttt{hunt})^n, (\neg\texttt{hunt}, \neg\texttt{cons})^n \Rightarrow \neg\texttt{cons}^o}\ \textit{Cut}$$

Two obligations: don't hunt of dragons (a) and don't ask for consent (b).

$\mathbb{K}_1$ does not support application of $(\texttt{hunt}, \texttt{cons})^n$ (since no hunt occurs).

KEEPING THE PROMISE TO HER BROTHER IN MIND, princess Charlotte decides to initiate a hunt for Albert. She remembers that she ought to be back on time for her brother's birthday though.

This a contrary-to-duty situation: A violation ensues.

The new knowledge base $\mathbb{K}_2$:

$$\mathcal{N} = \{(\top, \neg \mathtt{hunt})^n, (\neg \mathtt{hunt}, \neg \mathtt{cons})^n, (\mathtt{hunt}, \mathtt{cons})^n, (\mathtt{hunt}, \mathtt{back})^n\},$$
$$\mathcal{F} = \{\top^f, \mathtt{hunt}^f\},$$
$$\mathcal{C} = \{\top^c, \mathtt{hunt}^c\}.$$

The big question: What must Charlotte do given her violation?

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Part 5: CTD scenario

With $\mathbb{K}_2$ properly extending $\mathbb{K}_1$ we additionally derive:

$$c: \ \ \mathtt{hunt}^f, (\mathtt{hunt}, \mathtt{cons})^n \Rightarrow \mathtt{cons}^o \ \text{ and } \ d: \ \ \mathtt{hunt}^f, (\mathtt{hunt}, \mathtt{back})^n \Rightarrow \mathtt{back}^o$$

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With $\mathbb{K}_2$ properly extending $\mathbb{K}_1$ we additionally derive:

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We also have:

$$\dfrac{\dfrac{}{\top^c \Rightarrow \neg\bot^c}\,\mathit{Ax} \qquad \dfrac{\dfrac{b \qquad \dfrac{c \qquad \dfrac{}{\mathsf{cons}^o, \neg\mathsf{cons}^o \Rightarrow \bot^o}\,\mathit{Ax}}{\neg\mathsf{cons}^o, \mathsf{hunt}^f, (\mathsf{hunt}, \mathsf{cons})^n \Rightarrow \bot^o}\,\mathit{Cut}}{x:\ \top^f, \mathsf{hunt}^f, (\top, \neg\mathsf{hunt})^n, (\neg\mathsf{hunt}, \neg\mathsf{cons})^n, (\mathsf{hunt}, \mathsf{cons})^n \Rightarrow \bot^o}\,\mathit{Cut}}{\top^f, \mathsf{hunt}^f, (\top, \neg\mathsf{hunt})^n, (\neg\mathsf{hunt}, \neg\mathsf{cons})^n, (\mathsf{hunt}, \mathsf{cons})^n, \neg\bot^c \Rightarrow}\,\mathit{Cons}}{\dfrac{\top^f, \mathsf{hunt}^f, (\top, \neg\mathsf{hunt})^n, (\neg\mathsf{hunt}, \neg\mathsf{cons})^n, (\mathsf{hunt}, \mathsf{cons})^n, \top^c \Rightarrow}{e:\ \top^f, \mathsf{hunt}^f, (\neg\mathsf{hunt}, \neg\mathsf{cons})^n, (\mathsf{hunt}, \mathsf{cons})^n, \top^c \Rightarrow \neg(\top, \neg\mathsf{hunt})^n}\,\mathit{Inapp\ 3X}}\,\mathit{Cut}$$

and (dashed)

$$f:\ \top^f, \mathsf{hunt}^f, (\top, \neg\mathsf{hunt})^n, (\mathsf{hunt}, \mathsf{cons})^n, \top^c \Rightarrow \neg(\neg\mathsf{hunt}, \neg\mathsf{cons})^n$$

and (dashed)

$$g:\ \top^f, \mathsf{hunt}^f, (\top, \neg\mathsf{hunt})^n, (\neg\mathsf{hunt}, \neg\mathsf{cons})^n, \top^c \Rightarrow \neg(\mathsf{hunt}, \mathsf{cons})^n$$

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$\neg\mathsf{cons}^o$

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$$\dfrac{\top^f, \mathsf{hunt}^f, (\top, \neg\mathsf{hunt})^n, (\neg\mathsf{hunt}, \neg\mathsf{cons})^n, (\mathsf{hunt}, \mathsf{cons})^n, \top^c \Rightarrow}{e: \quad \top^f, \mathsf{hunt}^f, (\neg\mathsf{hunt}, \neg\mathsf{cons})^n, (\mathsf{hunt}, \mathsf{cons})^n, \top^c \Rightarrow \neg(\top, \neg\mathsf{hunt})^n}\,Inapp\ 3X$$

and

$$f: \quad \top^f, \mathsf{hunt}^f, (\top, \neg\mathsf{hunt})^n, (\mathsf{hunt}, \mathsf{cons})^n, \top^c \Rightarrow \neg(\neg\mathsf{hunt}, \neg\mathsf{cons})^n$$

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$$g: \quad \top^f, \mathsf{hunt}^f, (\top, \neg\mathsf{hunt})^n, (\neg\mathsf{hunt}, \neg\mathsf{cons})^n, \top^c \Rightarrow \neg(\mathsf{hunt}, \mathsf{cons})^n$$

With $\mathbb{K}_2$ properly extending $\mathbb{K}_1$ we additionally derive:

$$c: \ \mathsf{hunt}^f, (\mathsf{hunt}, \mathsf{cons})^n \Rightarrow \mathsf{cons}^o \quad \text{and} \quad d: \ \mathsf{hunt}^f, (\mathsf{hunt}, \mathsf{back})^n \Rightarrow \mathsf{back}^o$$

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Inapp 3X

$$e: \ \top^f, \mathsf{hunt}^f, (\neg\mathsf{hunt}, \neg\mathsf{cons})^n, (\mathsf{hunt}, \mathsf{cons})^n, \top^c \Rightarrow \neg(\top, \neg\mathsf{hunt})^n$$

and

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and

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!

The conflict between *e*, *f*, and *g* is due to general consistency $\top^c$.

Part 5: CTD scenario

We can **use the constraint** $\mathtt{hunt}^c$ in $\mathbb{K}_2$ to derive:

$$\dfrac{\dfrac{\dfrac{}{\mathtt{hunt}^c \Rightarrow \neg\neg\mathtt{hunt}^c}\,\mathit{Ax} \qquad \dfrac{a}{\top^f, (\top, \neg\mathtt{hunt})^n, \neg\neg\mathtt{hunt}^c \Rightarrow}\,\mathit{Cons}}{\top^f, (\top, \neg\mathtt{hunt})^n, \mathtt{hunt}^c \Rightarrow}\,\mathit{Cut}}{h :\ \ \top^f, \mathtt{hunt}^c \Rightarrow \neg(\top, \neg\mathtt{hunt})^n}\,\mathit{Inapp}$$

Hence: the norm $(\top, \neg\mathtt{hunt})^n$ becomes inapplicable given the violation!

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Hence: the norm $(\top, \neg\texttt{hunt})^n$ becomes inapplicable given the violation!

What to do with all these arguments? Formal argumentation!

- DAC-induced argumentation frameworks model conflicts!

Definition

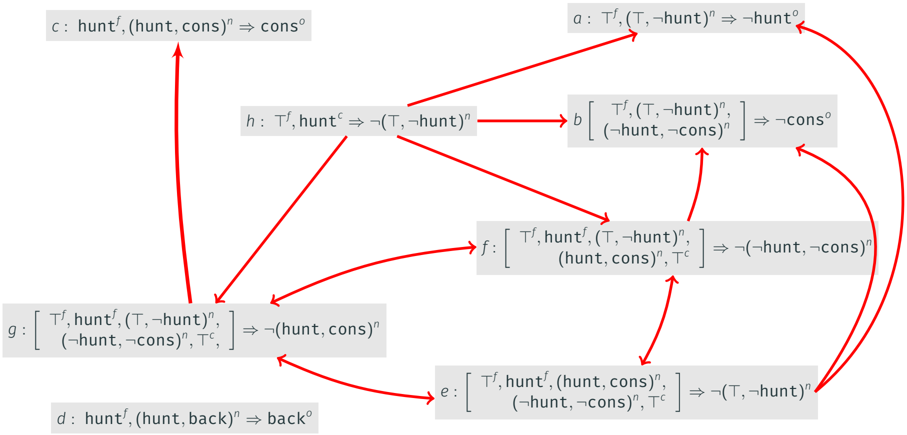
A DAC-induced Argumentation Framework:

Let $\mathbb{K} = \langle \mathcal{F}, \mathcal{N}, \mathcal{C} \rangle$ be a normative knowledge base. A DAC-induced argumentation framework is a tuple $\mathsf{AF}(\mathbb{K}) = \langle \mathsf{Arg}, \mathsf{Att} \rangle$ such that:

- $\Gamma \Rightarrow \Delta \in \mathsf{Arg}$ iff $\Gamma \Rightarrow \Delta$ is DAC-derivable and $\Gamma \subseteq \mathcal{F} \cup \mathcal{N} \cup \mathcal{C}$.

And for each $a, b \in \mathsf{Arg}$:

- *a attacks b*, i.e., $(a, b) \in \mathsf{Att}$ iff $a = \Gamma \Rightarrow \neg(\phi, \psi)$ and $b = \Delta, (\phi, \psi) \Rightarrow \Gamma$.



(Note that argument x concluding $\perp^o$ is omitted since it is attacked by all attackers)

Let's draw some conclusions!

Recall:

Definition

Stable:
$\mathcal{E}$ is stable iff it is conflict-free and for all $b \in \mathrm{Arg} \setminus \mathcal{E}$ there is a $c \in \mathcal{E}$ such that $(c, b) \in \mathrm{Attack}$.

Recall:

Definition

Stable:
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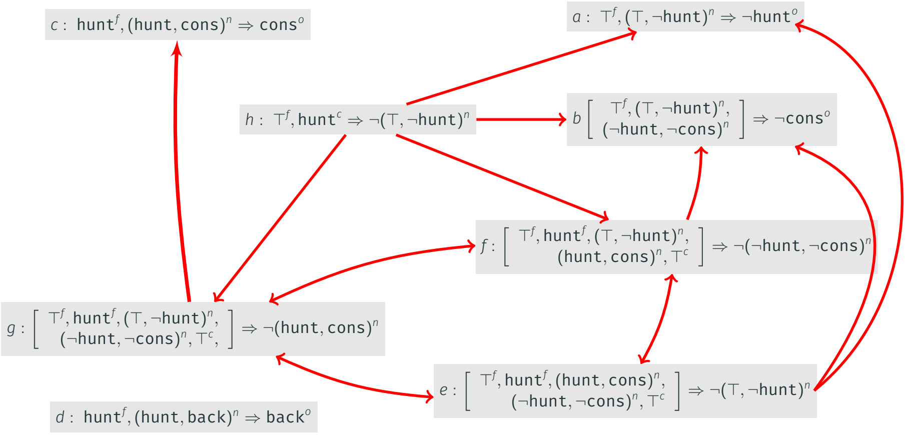
Definition

nm inference relations:

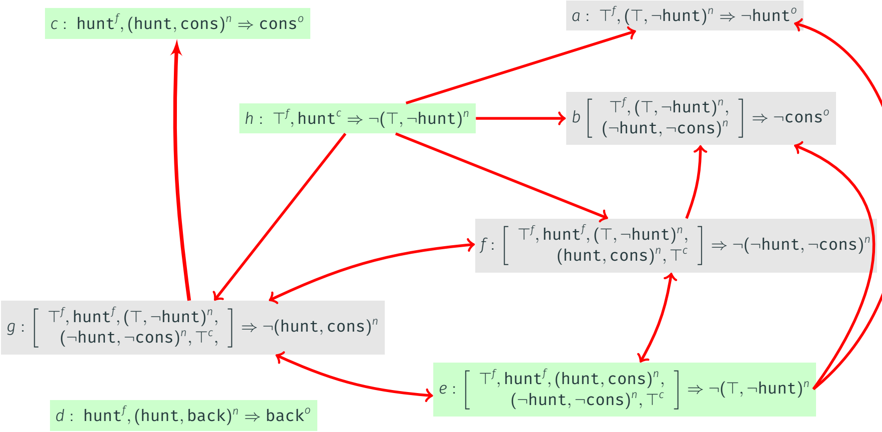
skeptical: $\mathbb{K} \mathrel{|\!\sim}^{\text{stable}}_{\cap\text{con}} \phi$ iff in every stable-extension $\mathcal{E}$ of $\text{AF}(\mathbb{K})$ there is an argument a with conclusion ϕ.

credulous: $\mathbb{K} \mathrel{|\!\sim}^{\text{stable}}_{\cup} \phi$ iff there is a stable-extension $\mathcal{E}$ of $\text{AF}(\mathbb{K})$ containing an argument a with conclusion ϕ.

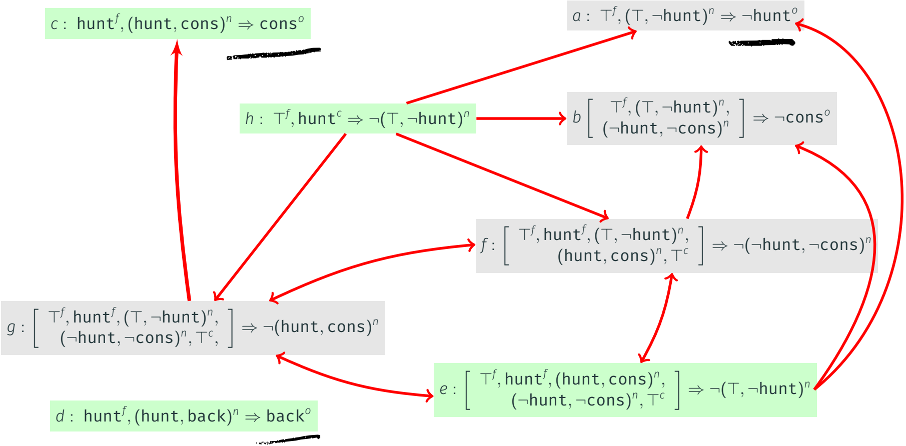
Part 5: DAC and AFs



Part 5: DAC and AFs



CTD Duties given $\mathbb{K}_2$ (with $\mathcal{F} = \{\top^f, \mathtt{hunt}^f\}$): Stable extension = $\{c, d, h, e\}$



CTD Duties given $\mathbb{K}_2$: $\mathbb{K}_2 \mathrel{|\!\sim}^{\text{stable}}_{\cap} \mathtt{cons}^o$, $\mathbb{K}_2 \mathrel{|\!\sim}^{\text{stable}}_{\cap} \mathtt{back}^o$, and $\mathbb{K}_2 \not\mathrel{|\!\sim}^{\text{stable}}_{\cap} \neg\mathtt{hunt}^o$

CTD Duties $(\mathbb{K}_2)$ with $\mathcal{F} = \{\top^f, \mathtt{hunt}^f\}$ skeptical inference:

$$\mathbb{K}_2 \mid\!\sim^{\text{stable}}_{\cap} \mathtt{cons}^o,\ \mathbb{K}_2 \mid\!\sim^{\text{stable}}_{\cap} \mathtt{back}^o,\ \text{ and } \mathbb{K}_2 \not\mid\!\sim^{\text{stable}}_{\cap} \neg\mathtt{hunt}^o$$

Note: the initial compliant situation $\mathbb{K}_1$ yields an AF with **different obligations**!

$$a:\ \top^f, (\top, \neg\mathtt{hunt})^n \Rightarrow \neg\mathtt{hunt}^o$$

$$b \left[\begin{array}{c} \top^f, (\top, \neg\mathtt{hunt})^n, \\ (\neg\mathtt{hunt}, \neg\mathtt{cons})^n \end{array} \right] \Rightarrow \neg\mathtt{cons}^o$$

Initial/Compliant duties $(\mathbb{K}_1)$ with $\mathcal{F} = \{\top^f\}$:

$$\mathbb{K}_1 \mid\!\sim^{\text{stable}}_{\cap} \neg\mathtt{hunt}^o,\ \mathbb{K}_1 \mid\!\sim^{\text{stable}}_{\cap} \neg\mathtt{cons}^o$$

Hence, extending $\mathbb{K}_1$ to $\mathbb{K}_2$ means **withdrawing obligations!**

The Input/Output (I/O) family (Makinson and van der Torre, 2000; 2001):

- Defeasible knowledge representation formalism for normative, causal, doxastic, legal reasoning,...

Theorem 1

Soundness and completeness result (van Berkel and Straßer, 2022):

DAC-instantiated AF($\mathbb{K}$)s are sound and complete with respect to its corresponding non-monotonic I/O logic:

- $\mathbb{K} \mid\!\sim^{\text{stable}}_{\cup/\cap} \phi^o$ iff ϕ is credulously/skeptical entailed in I/O logic.

This holds for 16 DAC and I/O systems.

Part 5: DAC results

The theorem contributes to the claim that formal argumentation is a **uniform formalism for NML**.

Such results are promising!

- We can use argumentation tools for this KRR framework!
- We can compare various NMLs in a shared setting.
- We can develop explainability methods for normative reasoning.

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- We can develop explainability methods for normative reasoning.

Similar results were obtained for Default Logic!

↖ see the talk by Zheng Zhou at Comma on Friday Sept 20.

Part 6: Let’s round up

Closing up

Key messages:

1. Defeasibility is ubiquitous: retracting conclusions.
2. Formal argumentation as a KR approach to defeasible reasoning: argument attack and selection.
3. Proof systems as a rule-based approach to generating arguments and refining different argument attacks.
4. Logical argumentation can satisfy various metatheoretic properties and rationality postulates are not guaranteed to hold (e.g., consistency!).
5. Application: logical argumentation provides more transparent reasoning with normative knowledge bases.

Closing up

Open problems and challenges:

1. NML with richer languages: preferences, FO, Modalities,....
2. Formal argumentation: Bridging the gap between symbolic and non-symbolic AI.
3. Automated reasoning: heuristics to work with finite AFs even when infinitely many arguments are available, etc.
4. Dialogues: construction of (deontic) explanation between humans and systems via argumentative exchange: many fields of research involved (NLP, ML, AF, philosophy).
5. . . .

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